Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 October/November 2024

Question No(10)

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A function f with domain $x > 0$ *is such* $\boldsymbol{that}\ \boldsymbol{f'(x) = 8(2x-3)^{\frac{1}{3}} - 10(x)^{\frac{2}{3}}}$. It is given that the curve with equation $y = f(x)$ passes through the point (1, 0).

(a) Find the equation of the normal to the curve at the point (1, 0).

(b) Find f(x).

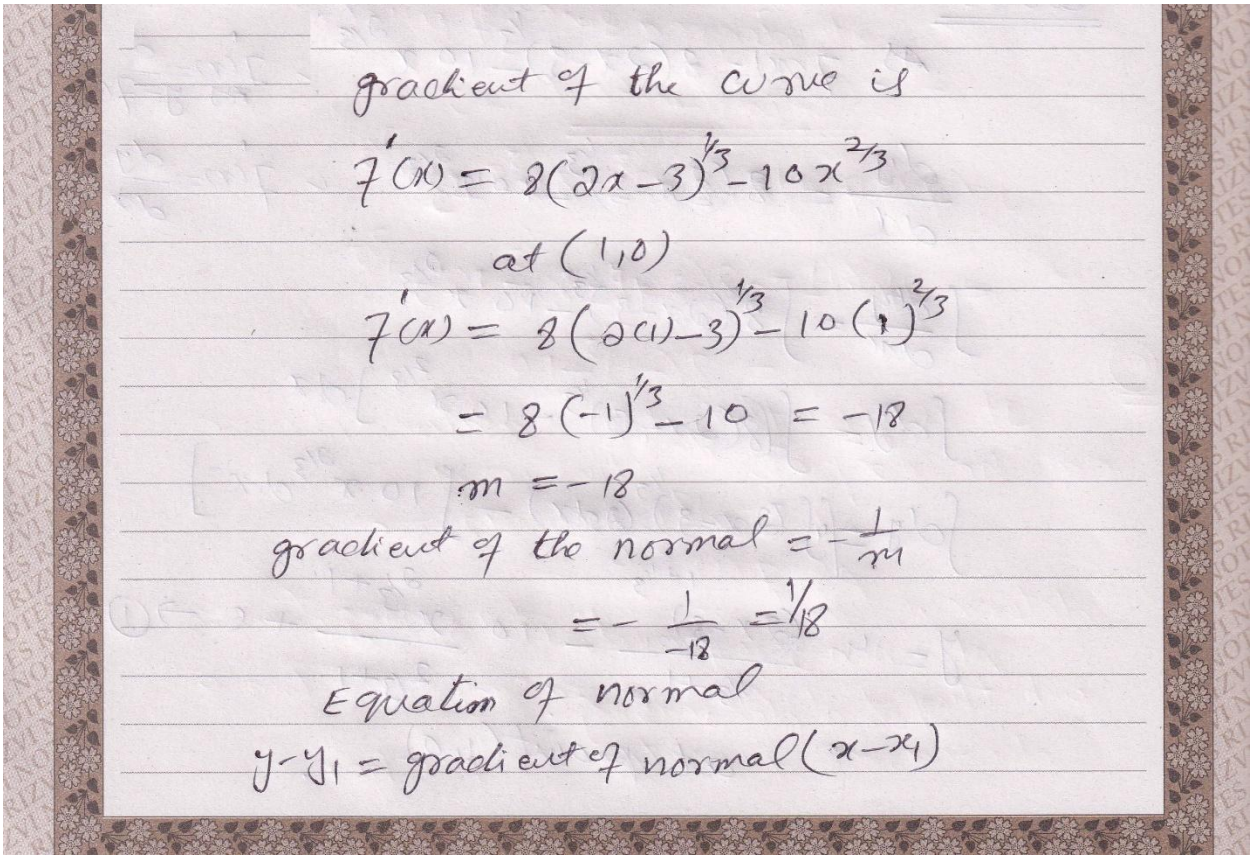
It is given that the equation $f'(x) = 0$, can be expressed in the form $\boldsymbol{125x^2 - 128x + 192 = 0}$.

(c) making your reasoning clear, whether f is an increasing function,

a decreasing function or neither.

Solution:

(a)



(34)

at point (1, 0)

Equation of normal come

$$y-0=\frac{1}{18}(x-1)$$

$$y=\frac{x}{18}-\frac{1}{18}$$

So Equation of normal is

$$y=\frac{x}{18}-\frac{1}{18}$$

(b) Find $f(x)$

Solution

As $f'(x)=8(2x-3)^{1/3}-10x^{2/3}$

As $y=f(x)$

$$\frac{dy}{dx}=8(2x-3)^{1/3}-10x^{2/3} \quad \therefore f'(x)=\frac{dy}{dx}$$

$$\int\frac{dy}{dx}=\int 8(2x-3)^{1/3}-10x^{2/3}$$

$$\int dy=\int\left[8(2x-3)^{1/3}-10x^{2/3}\right]dx$$

$$\int dy=\left[4\int(2x-3)^{1/3}(2dx)-\int 10x^{2/3}dx\right]$$

$$y=4\frac{(2x-3)^{1+1/3}}{1/3+1}-10\frac{x^{2/3+1}}{2/3+1}+c \rightarrow (1)$$

at point (1, 0)

DATE :- (35)

$$0 = \frac{4(-1)^{4/3}}{4/3} - 10\frac{(1)^{5/3}}{5/3} + c$$

$$0 = 4 \times 3/4 - 10 \times 3/5 + c$$

$$0 = 3 - 6 + c$$

$$c = 3$$

put $c = 3$ in Equation (1)

$$y = 4\frac{(2x-3)^{4/3}}{4/3} - 10\frac{x^{5/3}}{5/3} + 3$$

$$= 3(2x-3)^{4/3} - 6x^{5/3} + 3$$

$$y = f(x) = 3(2x-3)^{4/3} - 6x^{5/3} + 3$$

$$\Rightarrow f(x) = 3(2x-3)^{4/3} - 6x^{5/3} + 3$$

(c)

It is given that the equation $f'(x) = 0$ can be expressed in the form

$$125x^2 - 128x + 192 = 0$$

determine, making your reasoning clear, whether f is an increasing function, a decreasing function or neither.

(36)

Solution

Given

$125x^2 - 128x + 192 = 0$

$disc = b^2 - 4ac$

$= (128)^2 - 4(125)(192)$

$= 16384 - 96000$

$= -79616$

$\Rightarrow b^2 - 4ac < 0$

So its decreasing function.

second method

As $f'(x) = 8(2x-3)^{1/3} - 10x^{2/3}$

$f'(1) = 8(2(1)-3)^{1/3} - 10(1)^{2/3}$

$= 8(-1)^{1/3} - 10$

$= -8 - 10$

$= -18$

As $f'(1) < 0$

At non-zero point, it is negative, so this is decreasing.