

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

October/November 2024

Question No(4)

Question No (4)Find the term independent of x in the expansion of each of the following:

(a) $\left(x + \frac{3}{x^2}\right)^6$

(b) $(4x^3 - 5)\left(x + \frac{3}{x^2}\right)^6$

Solution:

(a)

$$\left(x + \frac{3}{x^2}\right)^6$$

Binomial expansion formula

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n b^n$$

$$\begin{aligned} \left(x + \frac{3}{x^2}\right)^6 &= {}^6C_0 x^6 + {}^6C_1 x^5 \left(\frac{3}{x^2}\right)^1 + {}^6C_2 x^4 \left(\frac{3}{x^2}\right)^2 \\ &\quad + {}^6C_3 x^3 \left(\frac{3}{x^2}\right)^3 + \dots + {}^6C_6 \left(\frac{3}{x^2}\right)^6 \end{aligned}$$

observe in above expansion. The term independent of x is

$${}^6C_2 x^4 \left(\frac{3}{x^2}\right)^2$$

$$\frac{6 \times 5}{2 \times 1} x^4 \cdot \frac{9}{x^4} = 135$$

DATE :-

$$\textcircled{b} \quad (4x^3 - 5) \left(x + \frac{3}{x^2}\right)^6 \quad \textcircled{11}$$

Solution

From part (a) we check that $\binom{6}{2} x^4 \left(\frac{3}{x^2}\right)^2$ has no x terms or independent of x , when we multiply it by 5 then again it will independent of x .

The second term in the expansion we need is

$$\binom{6}{3} x^3 \left(\frac{3}{x^2}\right)^3 \quad \text{when we shall}$$

multiply above term with $4x^3$, we shall get term independent of x , so

$$(4x^3 - 5) \left[\binom{6}{2} x^4 \left(\frac{3}{x^2}\right)^2 + \binom{6}{3} x^3 \left(\frac{3}{x^2}\right)^3 \right]$$

$$(4x^3 - 5) \left[\binom{6}{2} 9 + \binom{6}{3} x^3 \times \frac{27}{x^6} \right]$$

$$(4x^3 - 5) \left[\frac{6 \times 5 \times 9}{2 \times 1} + \frac{6 \times 5 \times 4}{3 \times 2 \times 1} x^3 \times \frac{27}{x^6} \right]$$

$$\frac{6 \times 5}{2 \times 1} \times 9x - 5 + \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4x^3 \times x^3 \times \frac{27}{x^6}$$

$$= -675 + 2160$$

$$= 1485$$