

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

October/November 2024

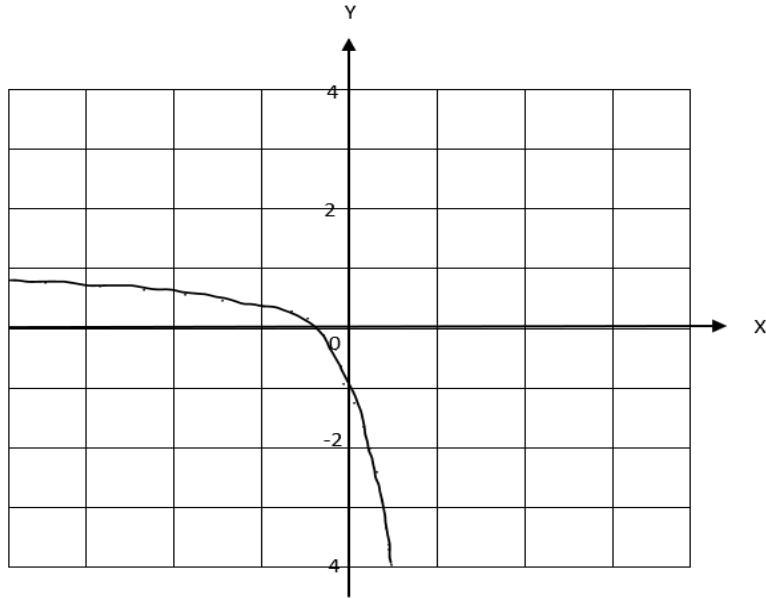
Question No(5)

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The function f is defined by $f(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$

(a) (i) State the value of $f(-1)$

(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1} .

The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$.

Solution

(a)(i)

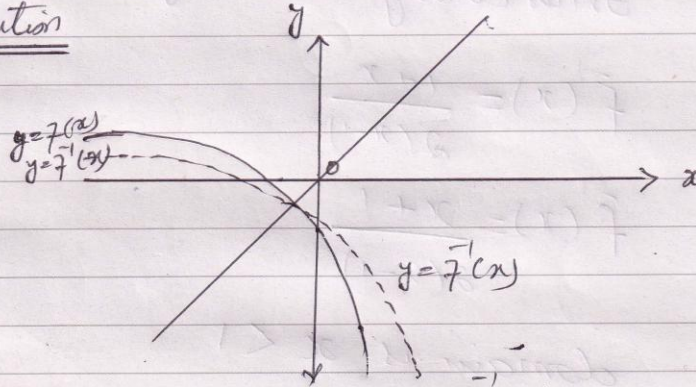
$$\begin{aligned}
 f(x) &= \frac{2x+1}{2x-1} \\
 f(-1) &= \frac{2(-1)+1}{2(-1)-1} \\
 &= \frac{-2+1}{-2-1} \\
 &= \frac{-1}{-3} \\
 f(-1) &= \frac{1}{3}
 \end{aligned}$$

(ii)

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The diagram shows the graph of $y = f(x)$.
 Sketch the graph of $y = f^{-1}(x)$ on this diagram.
 Show any relevant mirror line.

solution

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1} .

solution

$$f(x) = \frac{2x+1}{2x-1}$$

$$y = \frac{2x+1}{2x-1} \quad \checkmark \quad y = f(x)$$

$$y(2x-1) = 2x+1$$

$$2xy - y = 2x+1$$

$$2xy - 2x = 1+y$$

$$x(2y-2) = 1+y$$

$$x = \frac{1+y}{2(y-1)}$$

$$\bar{f}'(y) = \frac{1+y}{2(y-1)}$$

$$\begin{aligned} f(x) &= y \\ x &= \bar{f}'(y) \end{aligned}$$

interchange x and y

$$\bar{f}'(x) = \frac{1+x}{2(x-1)}$$

$$\bar{f}'(x) = \frac{x+1}{2(x-1)}$$

domain is $x < 1$

(b)

The function g is defined by $g(x) = 3x+2$ for $x \in \mathbb{R}$

Solve the equation $f(x) = g\bar{f}(1/4)$

solution

$$f(x) = g\bar{f}(1/4)$$

$$= g\left(\frac{2(1/4)+1}{2(1/4)-1}\right)$$

$$= g\left(\frac{1/2+1}{1/2-1}\right)$$

$$= g\left(\frac{3/2}{-1/2}\right)$$

$$= g(-3)$$

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$$= 3(-3) + 2$$

$$= -9 + 2$$

$$f(x) = -7$$

$$\Rightarrow \frac{2x+1}{2x-1} = -7$$

$$\therefore f(x) = \frac{2x+1}{2x-1}$$

$$2x+1 = -7(2x-1)$$

$$2x+1 = -14x+7$$

$$2x+14x = 7-1$$

$$16x = 6$$

$$x = \frac{6}{16}$$

$$x = \frac{3}{8}$$