Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 October/November 2024

Question No(7)

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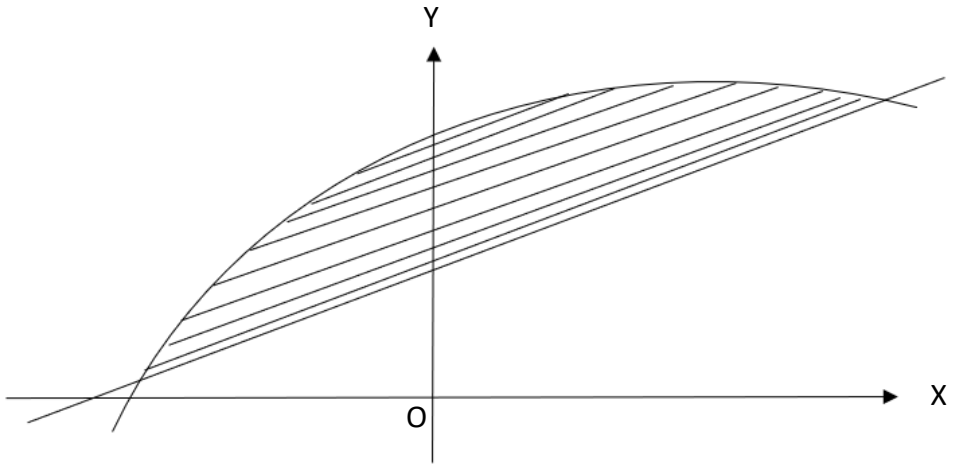
$(\boldsymbol{a})$ *By expressing* $-\mathbf{2}\,\boldsymbol{x^2} + \mathbf{8}\boldsymbol{x} + \mathbf{11}$ *in the form* $-\,a\,(x-b)^2 + c$,

where a, b *and* c *are positive integers* ,

find the coordinates of the vertex of the graph with equation $\boldsymbol{y} = -\mathbf{2}\,\boldsymbol{x^2} + \mathbf{8}\boldsymbol{x} + \mathbf{11}$

$(\boldsymbol{b})$



The diagram shows part of the curve with equation $\boldsymbol{y} = -\mathbf{2}\,\boldsymbol{x^2} + \mathbf{8}\boldsymbol{x} + \mathbf{11}$ and the line with equation $\boldsymbol{y} = \mathbf{8}\boldsymbol{x} + \mathbf{9}$.

Find the area of the shaded region.

Solution:

(a)

$-2x^2+8x+11$

$-2\left(x^2-\frac{8}{2}x\right)+11$

$-2\left(x^2-4x\right)+11$

$-2\left(x^2-2(2)x\right)+11$

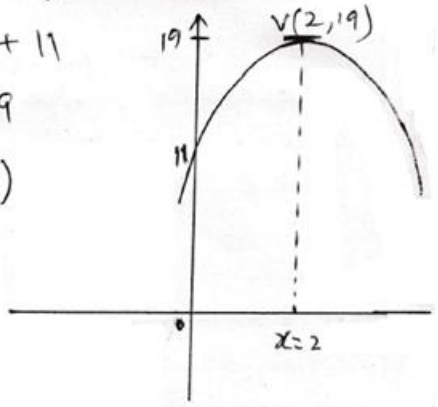
$-2\left(x^2-2(2)x+(2)^2-(2)^2\right)+11$

$-2\left(x^2-2(2)x+(2)^2\right)+2(2)^2+11$

$-2(x-2)^2+8+11$

$-2(x-2)^2+19$

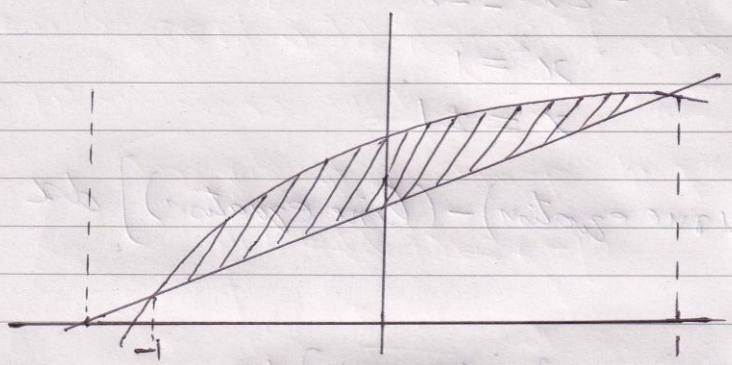
vertex $=(2,19)$



DATE :-

(21)

(b)



The diagram shows the part of the curve with equation $y = -2x^2 + 8x + 11$ and line with equation $y = 8x + 9$

Find the area of the shaded region.

Solution

As line is at the lower position than curve, so we shall subtract line from curve.

Formula for area

$$A = \int_a^b [(\text{Equation of curve}) - (\text{Equation of line})]\,dx$$

For limit, As the curve and line intersect, put $y = 8x + 9$ in curve

(22)

$$-2x^2+8x+11 = 8x+9$$

$$-2x^2 = -2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$A = \int_{-1}^{1} [(\text{curve equation}) - (\text{line equation})]\, dx$$

$$= \int_{-1}^{1} [(-2x^2+8x+11) - (8x+9)]\, dx$$

$$= \int_{-1}^{1} (-2x^2+8x+11-8x-9)\, dx$$

$$= \int_{-1}^{1} (-2x^2+2)\, dx$$

$$= \left| -\frac{2x^3}{3} + 2x \right|_{-1}^{1}$$

$$= -\frac{2(1)^3}{3} + 2(1) - \left(-\frac{2}{3}(-1)^3 + 2(-1)\right)$$

$$= -\frac{2}{3} + 2 - \frac{2}{3} + 2$$

$$= \frac{-2+6-2+6}{3} = \frac{12-4}{3}$$

$$A = 8/3$$