Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 October/November 2024

Question No(8)

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The equation of a circle is $x^2 + y^2 + px + 2y + q = 0$, where p and q are constants.

(a) Express the equation in the form $(x - a)^2 + (y - b)^2 = r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q.

The line with equation $x + 2y = 10$ is the tangent to the circle at the point A (4, 3).

(b) (i) Find the equation of the normal to the circle at the point A.

(ii) Find the values of p and q.

Solution:

(a)

$$x^2+y^2+px+2y+q=0$$

$$x^2+px+y^2+2y+q=0$$

$$x^2+\frac{2}{2}px+y^2+2(1)y+q=0$$

$$x^2+2\left(\frac{p}{2}\right)x+y^2+2(1)y+q=0$$

$$x^2+2(p/2)x+\left(\frac{p}{2}\right)^2-\left(\frac{p}{2}\right)^2+y^2+2(1)y+(1)^2-(1)^2+q=0$$

$$x^2+2(p/2)x+(p/2)^2+y^2+2(1)y+(1)^2=(p/2)^2-q+(1)^2$$

$$(x+p/2)^2+(y+1)^2=-q+1+(p/2)^2$$

$$(x-(-p/2))^2+(y-(-1))^2=-q+1+(-p/2)^2$$

(b) The line with equation $x+2y=10$ is the tangent to the circle at the point $A(4,3)$.

(i) Find the equation of the normal to the circle at the point A.

Solution

Given line

$$x+2y=10$$

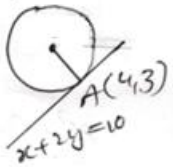
$$2y=-x+10$$

$$y=-\frac{1}{2}x+5$$

compare with y intercept form

$$y=mx+c$$

$$\Rightarrow m=-\frac{1}{2}$$



gradient of normal $=-\frac{1}{m}$

$$=-\frac{1}{-1/2}=2$$

Equation of normal at $A(4,3)$

$$y-y_1=2(x-x_1)$$

$$y-3=2(x-4)$$

$$y-3=2x-8$$

$$y=2x-5$$

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(ii) Find the values of p and q

Solution

Equation of circle

$$x^2+y^2+px+2y+q=0 \longrightarrow (1)$$

As, A(4,3) lie on circle, so

$$(4)^2+(3)^2+4p+6+q=0$$

$$16+9+4p+q+6=0$$

$$4p+q=-31 \longrightarrow (2)$$

gradient of the circle from equation (1)

Differentiate (1) w.r.t. x

$$2x+2y\frac{dy}{dx}+p+2\frac{dy}{dx}+0=0$$

$$(2y+2)\frac{dy}{dx}=-2x-p$$

$$\frac{dy}{dx}=-\frac{2x-p}{2y+2} \longrightarrow (3)$$

As the gradient of the circle is same as gradient of line, so equation (3) becomes

$$-\frac{2x-p}{2y+2}=-\frac{1}{2}$$

$\because -\frac{1}{2}$ gradient at circle

As, A(4, 3) lie on this, so

$$\frac{-2(4)-p}{2(3)+2} = -\frac{1}{2}$$

$$\frac{-8-p}{8} = -\frac{1}{2}$$

$$-8-p = -\frac{1}{2} \times 8 = -4$$

$$-p = -4+8$$

$$-p = 4$$

$$p = -4$$

put $p = -4$ in equation (2)

$$4(-4)+q = -31$$

$$q = -31+16$$

$$q = -15$$

So, $p = -4$, $q = -15$