Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 October/November 2024

Question No(9)

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The equation of a curve is $\boldsymbol{y = \frac{1}{2}k^2\,x^2 - 2kx + 2}$ and the equation of a line is $y = kx + p$, **where** k and p are constants with $0 < k < 1$.

(a) It is given that one of the points of intersection of the curve and the line has coordinates $\left(\frac{5}{2}, \frac{1}{2}\right)$. Find the values of k and p, and find the coordinates of the other point of intersection.

(b) It is given instead that the line and the curve do not intersect.
Find the set of possible values of p.

Solution:

(a)

Equation of the curve

$$y = \frac{1}{2}k^2x^2 - 2xk + 2 \longrightarrow (1)$$

Equation of line

$$y = kx + p \longrightarrow (2)$$

As the equation (1) and (2) intersect, so equate

$$\frac{1}{2}k^2x^2 - 2kx + 2 = kx + p$$

$$\frac{1}{2}k^2x^2 - 2kx + 2 - kx - p = 0$$

$\frac{1}{2}k^2x^2-3kx+2-p=0$

As point $(5/2, 1/2)$ lie on this, we get

$\frac{1}{2}k^2(5/2)^2-3k(5/2)+2-p=0$

$\frac{1}{2}k^2\left(\frac{25}{4}\right)-\frac{15k}{2}+2-p=0$

$25k^2-60k+16-8p=0 \longrightarrow (3)$

From equation (1)

$y=\frac{1}{2}k^2x^2-2kx+2$

As point $(5/2, 1/2)$ lie on this

$\frac{1}{2}=\frac{1}{2}k^2(5/2)^2-2k(5/2)+2$

$\frac{1}{2}=\frac{25}{8}k^2-\frac{10k}{2}+2$

$\frac{25}{8}k^2-\frac{10k}{2}+2-\frac{1}{2}=0$

$\frac{25}{8}k^2-5k+3/2=0$

$25k^2-40k+12=0$

By using quadratic formula

$a=25, \ b=-40, \ c=12$

$k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

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$$K = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(25)(12)}}{2 \times 25}$$

$$= \frac{40 \pm \sqrt{1600 - 1200}}{50}$$

$$= \frac{40 \pm \sqrt{400}}{50}$$

$$= \frac{40 \pm 20}{50}$$

$$= \left(\frac{40+20}{50}, \frac{40-20}{50}\right)$$

$$= \left(\frac{60}{50}, \frac{20}{50}\right)$$

$$K = \left(6/5, 2/5\right)$$

As $0 < K < 1$

$$\Rightarrow K = \frac{2}{5}$$

put $K = \frac{2}{5}$ in equation (3)

$$25\left(\frac{2}{5}\right)^2 - 60\left(\frac{2}{5}\right) + 16 - 8p = 0$$

$$25 \times \frac{4}{25} - \frac{120}{5} + 16 - 8p = 0$$

$$4 - \frac{120}{5} + 16 - 8p = 0$$

$$8P = 4 - \frac{120}{5} + 16$$

$$= \frac{20 - 120 + 80}{5}$$

$$8P = -4$$

$$P = -4/8$$

$$P = -1/2$$

As equation ① and ② intersect

$$\frac{1}{2}k^2x^2 - 2kx + 2 = kx + P$$

put $k = \frac{2}{5}$ and $P = -1/2$

$$\frac{1}{2}\left(\frac{2}{5}\right)^2 x^2 - 2\left(\frac{2}{5}\right)x + 2 = \frac{2}{5}x - \frac{1}{2}$$

$$\frac{1}{2} \times \frac{4}{25} x^2 - \frac{4}{5}x + 2 = \frac{2x}{5} - \frac{1}{2}$$

$$\frac{4}{50}x^2 - \frac{6}{5}x + \frac{5}{2} = 0$$

$$4x^2 - 60x + 125 = 0$$

using the quadratic formula

$$a = 4, b = -60, c = 125$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-60) \pm \sqrt{(-60)^2 - 4(4)(125)}}{2 \times 4}$$

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$$x = \frac{60 \pm \sqrt{3600 - 2000}}{8}$$

$$= \frac{60 \pm \sqrt{1600}}{8}$$

$$= \frac{60 \pm 40}{8}$$

$$= \left(\frac{60+40}{8}, \frac{60-40}{8}\right)$$

$$= \left(\frac{100}{8}, \frac{20}{8}\right)$$

$$x = \left(\frac{25}{2}, 5/2\right)$$

put $x = \frac{25}{2}$ in

$$y = kx + p$$

$$= \frac{2}{5}\left(\frac{25}{2}\right) - \frac{1}{2} \qquad \because k = \frac{2}{5}$$

$$p = -1/2$$

$$= \frac{50}{10} - \frac{1}{2}$$

$$= 5 - \frac{1}{2}$$

$$= \frac{10-1}{2}$$

$$y = 9/2$$

So, The second point is $\left(\frac{25}{2}, 9/2\right)$

(b) It is given instead that the line and the curve do not intersect.
Find the set of possible values of p

Solution
From the curve and line

$\frac{1}{2}k^2x^2 - 2kx + 2 = kx + p$

$\frac{1}{2}k^2x^2 - 3kx + 2 - p$

For the curve and line do not intersect

$b^2 - 4ac < 0$ disc $= b^2 - 4ac$

$(-3k)^2 - 4(\frac{1}{2}k^2)(2-p) < 0$

$9k^2 - 2k^2(2-p) < 0$

$9k^2 - 4k^2 + 2k^2p < 0$

$5k^2 + 2k^2p < 0$

$k^2(5+2p) < 0$

As k^2 is +ve, so

$5 + 2p < 0$

$2p < -5$

$p < -5/2$