Cambridge International AS & A Level

Mathematics

9709/32

Paper 3 Pure Mathematics 3 October/November 2024

Question No(10)

## Question No (10)

A balloon in the shape of a sphere has volume V and radius r. Air is pumped into the balloon at a constant rate of  $40\pi$  starting when time t = 0 and r =0. At the same time, air begins to flow out of the balloon at a rate of  $0.80~\pi r$ . The balloon remains a sphere at all times.

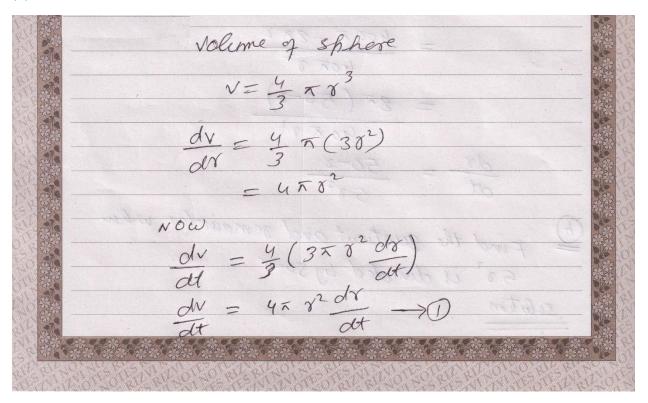
(a) Show that r and t satisfy the differential equation

$$\frac{dr}{dt} = \frac{50 - r}{5r^2}$$

- (b) Find the quotient and remainder when  $5r^2$  is divided by 50-r .
- (c) Solve the differential equation in part (a), obtaining an expression for t in terms of r.
- (d) Find the value of t when the radius of the balloon is 12.

## Solution:

(a)



As given in the statement of question vote et charge et volume

 $\frac{dv}{dt} = 40 \times -0.8 \times 8$  (AS -0.8 × 7 low)

From equation 
$$\frac{dx}{dt} = \frac{dx}{u \times x^2}$$

 $=\frac{40\pi-0.8\pi8}{4\pi8^2}$   $\frac{dV}{dt}=40\pi-0.8\pi8$ 

$$\frac{dx}{out} = \frac{40x - \frac{8}{10}x^{2}}{4xx^{2}}$$

$$= \frac{400x - 8xx}{40xx^{2}}$$

$$= \frac{8x(50-x)}{40xx^{2}}$$

$$\frac{ds}{dt} = \frac{50-8}{5s^2}$$

(b) Find the quotient and remainder when 582 is divided by 50-8 Solution

DATE:-	
DATE:-	(29)
	-58-250
VIII (I) CO	50-8 / 582
	1-582 = 2508
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	2508
	-2508 -12500 +12500
	12500
91	uotient = -58 - 250
nea	mainder = 12500
	put your and Cottleton
<b>a</b>	title-000 was energy and the
Sol1	an expression for t in terms of to.
aptaining	an expression for 4 in terms of 8.
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20 lution	The Battern 13.0
,	From (a)
2747	d8 _ 50-8
	$\frac{ds}{at} = \frac{50-s}{5s^2}$
033A) C+ + + -	separate the variables
	1387 10 10 10 10 10 10 10 10 10 10 10 10 10
-	separate the variables $dt = \frac{58}{50-8} d8$
	$dt = (-58 - 250 + \frac{12500}{50 - 8}) d8$
	dt = (-58-250+
	E son bost

 $\int dt = \int (-58 - 250 + \frac{12500}{50-8}) d8$  $t = -\frac{58^2}{3} - 2508 - 12500 \ln(50 - 0) + C \rightarrow 0$ whose cis constant of gotegration As given, at t=0, 820 above equation becomes 0=0-0-12500 ln50+e C= (2500 Puso put value of C=12500 luso in 1  $t = -\frac{5}{2} \delta^2 - 250 \delta - 12500 \ln(50 - 8) + 12500 \ln 50$ Find The value of t when the radius of The balloon is 12 Solution From past (C)  $t = -\frac{5}{2}r^2 - 2508 - 12500 \ln (50 - 8) + 12500 \ln 50$ at 8=12  $t = -\frac{5}{2}(12)^{2} - 250(12) - 12500 \ln(50 - 12) + 12500 \ln 50$ 

t = 70.5