Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 October/November 2024

Question No(5)

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(a) The complex number u is given by

$$u = \frac{(\cos\frac{\pi}{7} + i\ \sin\frac{\pi}{7})^4}{\cos\frac{\pi}{7} - i\ \sin\frac{\pi}{7}}$$

Find the exact value of arg u.

(b) The complex numbers u and u* are plotted on an Argand diagram.
Describe the single geometrical transformation that maps u onto u* and state the exact value of arg u*.

Solution:

$$u = \frac{(\cos \pi/7 + i \sin \pi/7)^4}{\cos \pi/7 - i \sin \pi/7}$$

$$= \frac{(\cos \pi/7 + i \sin \pi/7)^4}{\cos \pi/7 - i \sin \pi/7} \times \frac{\cos \pi/7 + i \sin \pi/7}{\cos \pi/7 + i \sin \pi/7}$$

$$= \frac{(\cos \pi/7 + i \sin \pi/7)^5}{\cos^2 \pi/7 - i^2 \sin^2 \pi/7}$$

$$= \frac{(\cos \pi/7 + i \sin \pi/7)^5}{\cos^2 \pi/7 + \sin^2 \pi/7} \qquad \because i^2 = -1$$

$$= (\cos \pi/7 + i \sin \pi/7)^5 \qquad \because \sin^2 \pi/7 + \cos^2 \pi/7 = 1$$

$$u = \cos \frac{5\pi}{7} + i \sin \frac{5\pi}{7} \qquad \text{By Demoviers theorem}$$

$$\arg u = \tan^{-1}\left(\frac{\sin 5\pi/7}{\cos 5\pi/7}\right)$$

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$$= \tan^{-1}\left(\tan \frac{5\pi}{7}\right)$$

$$\arg u = 5\pi/7$$

(b)

The complex numbers u and u^* are plotted on an Argand diagram. Describe the single geometrical transformation that maps u and u^* and state the exact value of $\arg u^*$.

Solution

As value of u from (a) is

$$u = \cos \frac{5\pi}{7} + i \sin \frac{5\pi}{7}$$

$$z = \cos \frac{5\pi}{7} + i \sin \frac{5\pi}{7}$$

$$|z| = \sqrt{\left(\cos \frac{5\pi}{7}\right)^2 + \left(\sin \frac{5\pi}{7}\right)^2}$$

$$= \sqrt{\cos^2 \frac{5\pi}{7} + \sin^2 \frac{5\pi}{7}}$$

$$= \sqrt{1}$$

$$|z| = 1$$

$$\arg z = \tan^{-1}\left(\frac{\sin 5\pi/7}{\cos 5\pi/7}\right) = \tan^{-1}\left(\tan \frac{5\pi}{7}\right)$$

$$= \frac{5\pi}{7}$$

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Now

$$u^* = \cos\frac{5\pi}{7} - i\sin\frac{5\pi}{7}$$ conjugate of u

$$\bar{z} = \cos\frac{5\pi}{7} - i\sin\frac{5\pi}{7}$$

if $z = x+iy$
$\bar{z} = x-iy$

$$|\bar{z}| = \sqrt{\left(\cos\frac{5\pi}{7}\right)^2 + \left(-\sin\frac{5\pi}{7}\right)^2}$$

$$= \sqrt{\cos^2\frac{5\pi}{7} + \sin^2\frac{5\pi}{7}}$$

$$= \sqrt{1}$$ $\because \sin^2\theta + \cos^2\theta = 1$

$$= 1$$

$$\arg u^* = \tan^{-1}\left(-\frac{\sin 5\pi/7}{\cos 5\pi/7}\right)$$

$$= \tan^{-1}\left(\tan\left(-\frac{5\pi}{7}\right)\right)$$

$$\arg u^* = -\frac{5\pi}{7}$$

From the argument of u and u^* its a Transformation of type reflection in Real axis