Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 October/November 2024

Question No(7)

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(a) Show that the equation $tan^3x + 2\tan 2x - tanx = 0$ may be expressed as

$$tan^4x - 2\,tan^2x\ -3 = 0$$

For $tanx\ \neq 0$.

(b) Hence solve the equation $tan^3 2\theta + 2\tan 4\theta - tan2\theta = 0$ for $0 < \theta < \pi$. Give your answers in exact form.

Solution:(a)

Given equation

$$\tan^3 x + 2\tan 2x - \tan x = 0 \rightarrow (1)$$

Trigonometric formula

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Equation (1) becomes

$$\tan^3 x + 2\left(\frac{2\tan x}{1-\tan^2 x}\right) - \tan x = 0$$

$$\tan^3 x + \frac{4\tan x}{1-\tan^2 x} - \tan x = 0$$

$$\tan^3 x(1-\tan^2 x) + 4\tan x - \tan x(1-\tan^2 x) = 0$$

$$\tan^3 x - \tan^5 x + 4\tan x - \tan x + \tan^3 x = 0$$

(14)

$$-\tan^5 x + 2\tan^3 x + 3\tan x = 0$$

$$-\tan x(\tan^4 x - 2\tan^2 x - 3) = 0$$

As $\tan x \neq 0$

$$\Rightarrow \tan^4 x - 2\tan^2 x - 3 = 0$$

(b) Hence solve the equation $\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0$ for $0 < \theta < \pi$.

Give your answer in exact form.

Solution

$$\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0, \quad 0 < \theta < \pi$$

put $2\theta = x$

$$\tan^3 x + 2\tan 2x - \tan x = 0, \quad 0 < x < 2\pi$$

above equation becomes

$$\tan^4 x - 2\tan^2 x - 3 = 0 \quad \text{from (a)}$$

By factorization

$$\tan^4 x - 3\tan^2 x + \tan^2 x - 3 = 0$$

$$\tan^2 x(\tan^2 x - 3) + 1(\tan^2 x - 3) = 0$$

$$(\tan^2 x - 3)(\tan^2 x + 1) = 0$$

$0 < \theta < \pi$

$0 \times 2 < 2\theta < 2\pi$

$0 < 2\theta < 2\pi$

$0 < x < 2\pi$

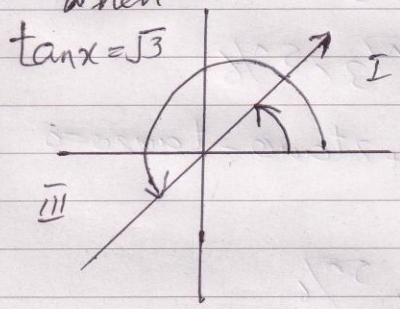
DATE :- (15)

$\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

when $\tan x = \sqrt{3}$



Basic angle

$\alpha = \tan^{-1}(\sqrt{3})$

$\alpha = \pi/3$

$\alpha, \quad \pi + \alpha$

$\pi/3, \quad \pi + \pi/3$

$\pi/3, \quad 4\pi/3$

So $x = \pi/3, 4\pi/3$

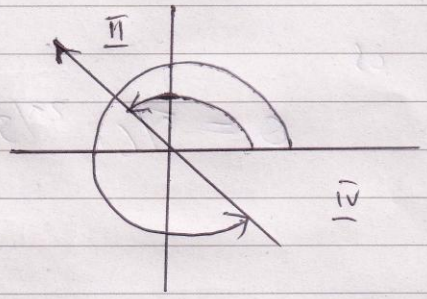
$\tan^2 x + 1 = 0$

$\tan^2 x = -1$

$\tan x = \pm\sqrt{-1}$

$\tan x = \pm i$ (ignore)

when $\tan x = -\sqrt{3}$



Basic angle

$\alpha = \tan^{-1}(\sqrt{3})$

$\alpha = \pi/3$

$\pi - \alpha, \quad 2\pi - \alpha$

$\pi - \pi/3, \quad 2\pi - \pi/3$

$2\pi/3, \quad 5\pi/3$

$x = 2\pi/3, 5\pi/3$

(16)

So the values of x are

$$x = \pi/3, 4\pi/3, 2\pi/3, 5\pi/3$$

As $x = 2\theta$

$$\Rightarrow 2\theta = \pi/3, 4\pi/3, 2\pi/3, 5\pi/3$$

$$\theta = \pi/6, 2\pi/3, \pi/3, 5\pi/6$$

Hence solution for $\tan^3 2\theta + 2\tan\theta - \tan 2\theta = 0$ is

$$\theta = \pi/6, 2\pi/3, \pi/3, 5\pi/6$$