Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 October/November 2024

Question No(8)

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The parametric equations of a curve are

$$x = tan^2 2t, \qquad y = cos2t,$$

$for\ 0 < t < \frac{\pi}{4}$

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\, cos^3\, 2t.$

(b) Hence find the equation of the normal to the curve at the point where $t = \frac{\pi}{8}$. Give your answer in the form $y = mx + c$.

Solution:

$$x = \tan^2 2t$$

$$\frac{dx}{dt} = 2\tan 2t(\sec^2 2t)(2)$$

$$= 4\tan 2t \sec^2 2t$$

$$\frac{dx}{dt} = 4\,\frac{\sin 2t}{\cos 2t}\cdot\frac{1}{\cos^2 2t}$$

$$y = \cos 2t$$

$$\frac{dy}{dt} = -\sin 2t(2)$$

$$= -2\sin 2t$$

Now

$$\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$$

$$= -2\sin 2t \times \frac{\cos 2t \times \cos^2 2t}{4\sin 2t}$$

$$\frac{dy}{dx} = -\frac{\cos^3 2t}{2}$$

(18)

(b) Hence find the equation of the normal to the curve at the point where $t = \pi/8$. Give your answer in the form $y = mx + c$

Solution

From part (a)

$$\frac{dy}{dx} = -\frac{\cos^3 2t}{2}$$

$$= -\frac{\cos^3 2(\pi/8)}{2}$$

$$= -\frac{\cos^3 \frac{\pi}{4}}{2}$$

$$= -\frac{(\cos \pi/4)^3}{2}$$

$$= -\frac{(1/\sqrt{2})^3}{2}$$

$$= -\frac{1}{\sqrt{2}\sqrt{2}\sqrt{2}\ 2}$$

$$\frac{dy}{dx} = -\frac{1}{4\sqrt{2}}$$

gradient of normal $= -\frac{1}{\frac{dy}{dx}}$

$$= -\frac{1}{-\frac{1}{4\sqrt{2}}} = 4\sqrt{2}$$

DATE :- (19)

As $x = \tan^2 2t$, $y = \cos 2t$

$x = \tan^2 2(\pi/8)$	$y = \cos 2(\pi/8)$
$= \tan^2 \pi/4$	$= \cos \pi/4$
$= (\tan \pi/4)^2$	$= \frac{1}{\sqrt{2}}$
$= (1)^2$	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$x = 1$	$y = \frac{\sqrt{2}}{2}$

Equation of normal

$y - y_1 = \text{gradient of normal } (x - x_1)$

$$y - \frac{\sqrt{2}}{2} = 4\sqrt{2}(x-1)$$

$$y = \frac{\sqrt{2}}{2} + 4\sqrt{2}x - 4\sqrt{2}$$

$$= 4\sqrt{2}x - 4\sqrt{2} + \frac{\sqrt{2}}{2}$$

$$= 4\sqrt{2}x - \frac{8\sqrt{2} + \sqrt{2}}{2}$$

$$y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$$

required Form.