Cambridge International AS & A Level

Mathematics 9709/42

Paper 4 Mechanics October/November 2024

Question No(5)

Question No (5)

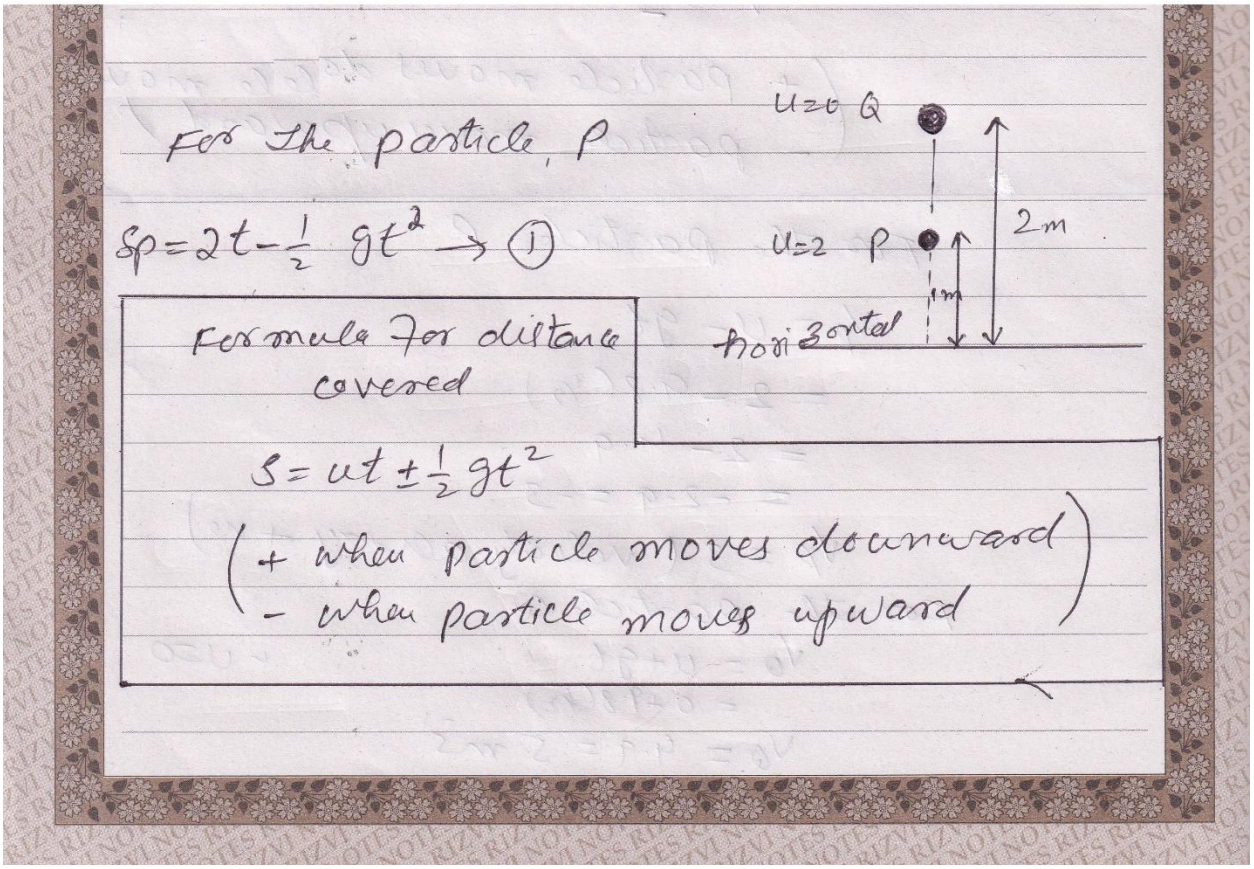
Two particles, P and Q, of masses 2m kg and m kg respectively, are held at rest in the same vertical line. The heights of P and Q above horizontal ground are 1 m and 2 m respectively. P is projected vertically upwards with speed 2 m s^{-1}. At the same instant, Q is released from rest.

(a) Find the speed of each particle immediately before they collide.

(b) It is given that immediately after the collision the downward speed of Q is 3.5 ms^{-1}. Find the speed of P at the instant that it reaches the ground.

Solution:

(a)



For the particle Q

$$s_Q = \frac{1}{2} g t^2 \longrightarrow (2)$$

(sum of distance of two particles)

As $s_P + s_Q = 2 - 1$

$$2t - \frac{1}{2} g t^2 + \frac{1}{2} g t^2 = 1$$

$$2t = 1$$

$$t = 1/2$$

Equation of motion

$$v = u \pm at$$

(+ particle moves downward
− particle moves upward)

For the particle P

$$v_P = u - gt$$

$$= 2 - 9.8(1/2)$$

$$= 2 - 4.9$$

$$= -2.9 = -3$$

$v_P = 3$ (velocity always +ve)

For the particle Q

$$v_Q = u + gt \qquad \because u = 0$$

$$= 0 + 9.8(1/2)$$

$$v_Q = 4.9 = 5\ m\,s^{-1}$$

(b)

By Law of conservation of momentum

initial momentum = final momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2m(3) + m(5) = 2mv + m(3.5)$

$6m + 5m = 2mv + 3.5m$

$m(6+5) = m(2v + 3.5)$

$6+5 = 2v + 3.5$

$11 = 2v + 3.5$

$v = \frac{11 - 3.5}{2}$

$v = 3.7$

So after collision velocity of P is 3.7 m/s

At the time of collision both particles are at same level from the horizontal.

so at $t = 1/2$

distance for particle P is

$= 1 + ut - \frac{1}{2}gt^2$

$S_p(1/2) = 1 + 2(1/2) - \frac{1}{2}(9.8)(1/2)^2$

$= 1 + 1 - 1.225$

$= 2 - 1.225$

$S_p(1/2) = 0.775$ m

$S_p = 0.775$

For the particle P, velocity before hitting ground

$u = 3.7$ m/s (initial velocity after collision)

$g = 9.8$ m/s^2

$S_p = 0.775$

$g = 9.8$ m/s^2

using the equation

$v^2 - u^2 = 2as$

$v^2 - (3.7)^2 = 2(9.8)(0.775)$

$v^2 = (3.7)^2 + 2(9.8)(0.775)$

$v^2 = 28.88$

$v = 5.37$ m/s