Cambridge International AS & A Level

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Mathematics 9709/42

Paper 4 Mechanics October/November 2024

Question No(6)

**Question No (6)**

A particle, P, travels in a straight line, starting from a point O with velocity 6 $ms^{-1}$. The acceleration of P at time t s after leaving O is a $ms^{-2}$, where

$$a = -1.5\,t^{\frac{1}{2}} \quad for\ 0 \leq t \leq 1,$$

$$a = 1.5\,t^{\frac{1}{2}} - 3\,t^{\frac{-1}{2}} \quad for\ t > 1.$$

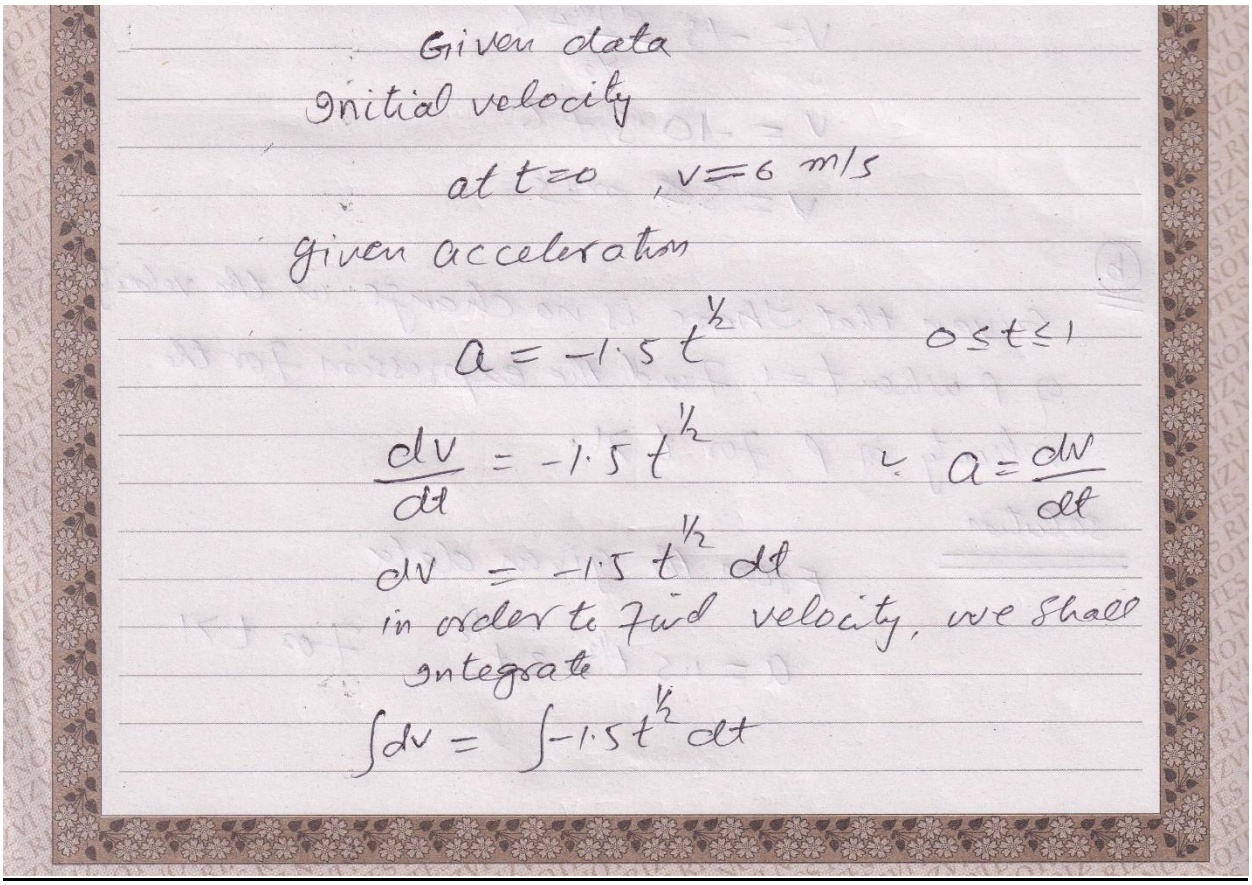
(a) Find the velocity of P at t =1 .

(b) Given that there is no change in the velocity of P when t =1 , find an expression for the velocity of P for $t > 1$ .

(c) Given that the velocity of P is positive for $t \leq 4$ , find the total distance travelled between t =0 and t= 4.

Solution:

(a)



$$v = -1.5\frac{t^{1/2+1}}{1/2+1} + c \rightarrow (1) \quad (c \text{ constant of integration})$$

at $t = 0$, $v = 6$

$$\Rightarrow 6 = \frac{-1.5(0)^{3/2}}{3/2} + c$$

$$\Rightarrow c = 6$$

put $c = 6$ in (1)

$$v = -1.5\frac{t^{3/2}}{3/2} + 6$$

in order to find velocity at $t = 1$

$$v = -1.5\frac{(1)^{3/2}}{3/2} + 6$$

$$v = -1 + 6$$

$$v = 5 \text{ m/s}$$

(b)

From the given data

$$a = 1.5t^{1/2} - 3t^{-1/2} \quad \text{for } t > 1$$

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$$\frac{dv}{dt} = 1.5t^{1/2} - 3t^{-1/2}$$

$$dv = (1.5t^{1/2} - 3t^{-1/2})\,dt$$

Integrate

$$\int dv = \int (1.5t^{1/2} - 3t^{-1/2})\,dt$$

$$v = 1.5\frac{t^{1/2+1}}{1/2+1} - 3\frac{t^{-1/2+1}}{-1/2+1} + c \rightarrow (1)$$

As velocity remains same after $t = 1$,

As at $t = 1$, $v = 5$ from part (a)

equation (1) becomes

$$5 = 1.5\frac{(1)^{3/2}}{3/2} - 3\frac{(1)^{1/2}}{1/2} + c$$

$$5 = 1 - 6 + c$$

$$c = 10$$

put $c = 10$ in (1)

$$v = 1.5\frac{t^{3/2}}{3/2} - 3\frac{t^{1/2}}{1/2} + 10$$

$$v = t^{3/2} - 6t^{1/2} + 10$$

(c)

From the part (a) The velocity in the time $0 \to 1$ is

$$v = -1.5 \frac{t^{3/2}}{3/2} + 6$$

$$= -1.5 \times \frac{2}{3} t^{3/2} + 6$$

$$v = -t^{3/2} + 6$$

$$\frac{ds}{dt} = -t^{3/2} + 6$$

$$ds = (-t^{3/2} + 6)\,dt$$

Integrate

$$\int ds = \int_0^1 (-t^{3/2} + 6)\,dt$$

$$= \left| -\frac{t^{3/2+1}}{3/2+1} + 6t \right|_0^1$$

$$= \left| -\frac{t^{5/2}}{5/2} + 6t \right|_0^1$$

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$$= \frac{-4)^{5/2}}{5/2} + 6(1) - [-0+0]$$

$$= -\frac{2}{5} + 6$$

$$= -0.4 + 6$$

$$S_1 = 5.6$$

Now we find the distance from time $t=1 \to t=4$ from part (b)

$$v = t^{3/2} - 6t^{1/2} + 10$$

$$\frac{ds}{dt} = t^{3/2} - 6t^{1/2} + 10$$

$$ds = (t^{3/2} - 6t^{1/2} + 10)\,dt$$

integrate

$$s = \int_1^4 (t^{3/2} - 6t^{1/2} + 10)\,dt$$

$$= \left| \frac{t^{3/2+1}}{3/2+1} - \frac{6t^{1/2+1}}{1/2+1} + 10t \right|_1^4$$

$$= \left| \frac{t^{5/2}}{5/2} - 6\frac{t^{3/2}}{3/2} + 10t \right|_1^4$$

$$= \left| \frac{2}{5} t^{5/2} - 4t^{3/2} + 10t \right|_1^4$$

$$= \frac{2}{5}(4)^{5/2} - 4(4)^{3/2} + 10(4) - \left[\frac{2}{5}(1)^{5/2} - 4(1)^{3/2} + 10(1)\right]$$

$$= \frac{2}{5}(2^2)^{5/2} - 4(2^3)^{3/2} + 40 - \left[\frac{2}{5} - 4 + 10\right]$$

$$= \frac{2}{5}(2)^5 - 4(2)^3 + 40 - \left[\frac{2}{5} - 4 + 10\right]$$

$$= \frac{2}{5}(32) - 4(8) + 40 - \frac{2}{5} + 4 - 10$$

$$= 12.8 - 32 + 40 - 0.4 + 4 - 10$$

$$S_2 = 14.1$$

total distance travelled from 0 to 4 is

$$S = S_1 + S_2$$

$$5.6 + 14.1$$

$$= 19.7$$

$$S = 20 \text{ m}$$