

Cambridge International AS & A Level

Mathematics

9709/52

Paper 5 Probability & Statistics 1

October/November 2024

Question No(3)

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Question No (3)

A fair coin and an ordinary fair six-sided dice are thrown at the same time. The random variable X is defined as follows.

- If the coin shows a tail, X is twice the score on the dice.
- If the coin shows a head, X is the score on the dice if the score is even and X is 0 otherwise.

(a) Draw up the probability distribution table for X .

b) Find $\text{Var}(X)$.

Solution:

(a)

Given data

coin $\rightarrow H, T$

dice $\rightarrow 1, 2, 3, 4, 5, 6$

According to given condition

if coin shows Tail, T , ~~dice~~ sides $1, 2, 3, 4, 5, 6$
becomes $2, 4, 6, 8, 10, 12$

if coin shows H , dice side $2, 4, 6$ remain
same and side $1, 3, 5$ becomes 0 .

DATE :-

(9)

sample space
dice

	1	2	3	4	5	6
H	0	2	0	4	0	6
T	2	4	6	8	10	12

probability distribution table for x.

x	0	2	4	6	8	10	12
$P(X=x)$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

(b) Find variance(X)

solutionFor var(X), we shall calculate
 $E(X)$ first

Formula

$$E(X) = \sum_i x_i \cdot P(X=x_i)$$

$$E(X) = 0\left(\frac{3}{12}\right) + 2\left(\frac{2}{12}\right) + 4\left(\frac{2}{12}\right) + 6\left(\frac{2}{12}\right) + 8\left(\frac{1}{12}\right) + 10\left(\frac{1}{12}\right) + 12\left(\frac{1}{12}\right)$$

$$\begin{aligned}
 E(X) &= \frac{4}{12} + \frac{8}{12} + \frac{12}{12} + \frac{8}{12} + \frac{10}{12} + \frac{12}{12} \\
 &= \frac{54}{12} \\
 E(X) &= 4.5
 \end{aligned}$$

Formula for

$$\text{var}(X) = \sum_i x_i^2 \cdot P(X=x_i) - [E(X)]^2$$

$$\begin{aligned}
 \text{var}(X) &= (0)^2 \frac{3}{12} + (2)^2 \frac{2}{12} + (4)^2 \frac{2}{12} + (6)^2 \frac{2}{12} + (8)^2 \frac{1}{12} + (10)^2 \left(\frac{1}{12}\right) + (12)^2 \left(\frac{1}{12}\right) - (4.5)^2 \\
 &= \frac{8}{12} + \frac{32}{12} + \frac{72}{12} + \frac{64}{12} + \frac{100}{12} + \frac{144}{12} - 20.25 \\
 &= 14.75
 \end{aligned}$$

$$\text{var}(X) = 14.75$$