

Cambridge International AS & A Level

Mathematics

9709/52

Paper 5 Probability & Statistics 1

October/November 2024

Question No(4)

<https://kingcambridgesolutions.com>

Question No (4)

The heights, in metres, of white pine trees are normally distributed with mean 19.8 and standard deviation 2.4.

In a certain forest there are 450 white pine trees.

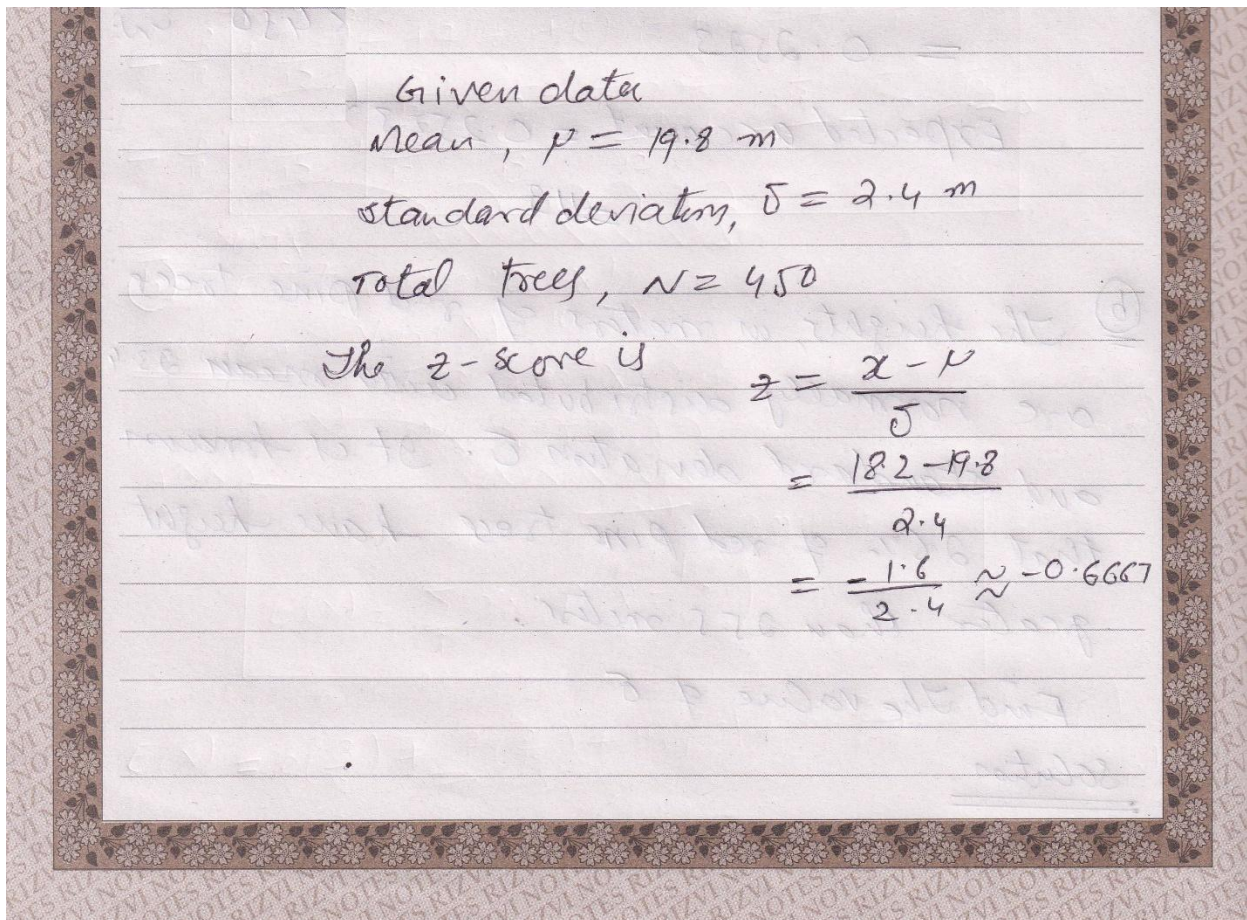
(a) How many of these trees would you expect to have height less than 18.2 metres?

The heights, in metres, of red pine trees are normally distributed with mean 23.4 and standard deviation σ . It is known that 26% of red pine trees have height greater than 25.5 metres.

(b) Find the value of σ .

Solution:

(a)



Given data
Mean, $\mu = 19.8$ m
standard deviation, $\sigma = 2.4$ m
total trees, $N = 450$

The z-score is
$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{18.2 - 19.8}{2.4}$$

$$= \frac{-1.6}{2.4} \approx -0.6667$$

According to the condition

$$P(X < 18.2)$$

$$P\left(z < \frac{18.2 - 19.8}{2.4}\right)$$

$$P(z < -0.6667)$$

$$= \Phi(-0.6667)$$

$$= 1 - \Phi(0.6667)$$

$$= 1 - 0.7477 \quad (\text{By ND table})$$

$$= 0.2523$$

By normal
distribution

$$\begin{aligned} \text{Expected account} &= 0.2523 \times 450 \quad \because N=450 \\ &= 113 \end{aligned}$$

(b)

DATE:-

(13)

Given data

Mean, $\mu = 23.4$ standard deviation, $\sigma = ?$

Given condition

$$P(X > 25.5) = 26\%$$

$$P(X > 25.5) = 0.26$$

$$P(X < 25.5) = 1 - 0.26$$

$$P(X < 25.5) = 0.74$$

$$P\left(Z < \frac{X - \mu}{\sigma}\right) = 0.74$$

$$P\left(Z < \frac{25.5 - 23.4}{\sigma}\right) = 0.74$$

$$P\left(Z < \frac{25.5 - 23.4}{\sigma}\right) = 0.74$$

$$P\left(Z < \frac{2.1}{\sigma}\right) = 0.74$$

$$\Phi\left(\frac{2.1}{\sigma}\right) = 0.74$$

$$\frac{2.1}{\sigma} = \Phi^{-1}(0.74)$$

$$= 0.643$$

$$\sigma = 3.27$$

