Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2024**

Question No (10)

https://kingcambridgesolutions.com

Question No (10)

The equation of a curve is $y = (5 - 2x)^{\frac{3}{2}} + 5 \ for\, x < \frac{5}{2}$.

(a) A point P is moving along the curve in such a way that the y-coordinate of point P is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is increasing when y =32 .

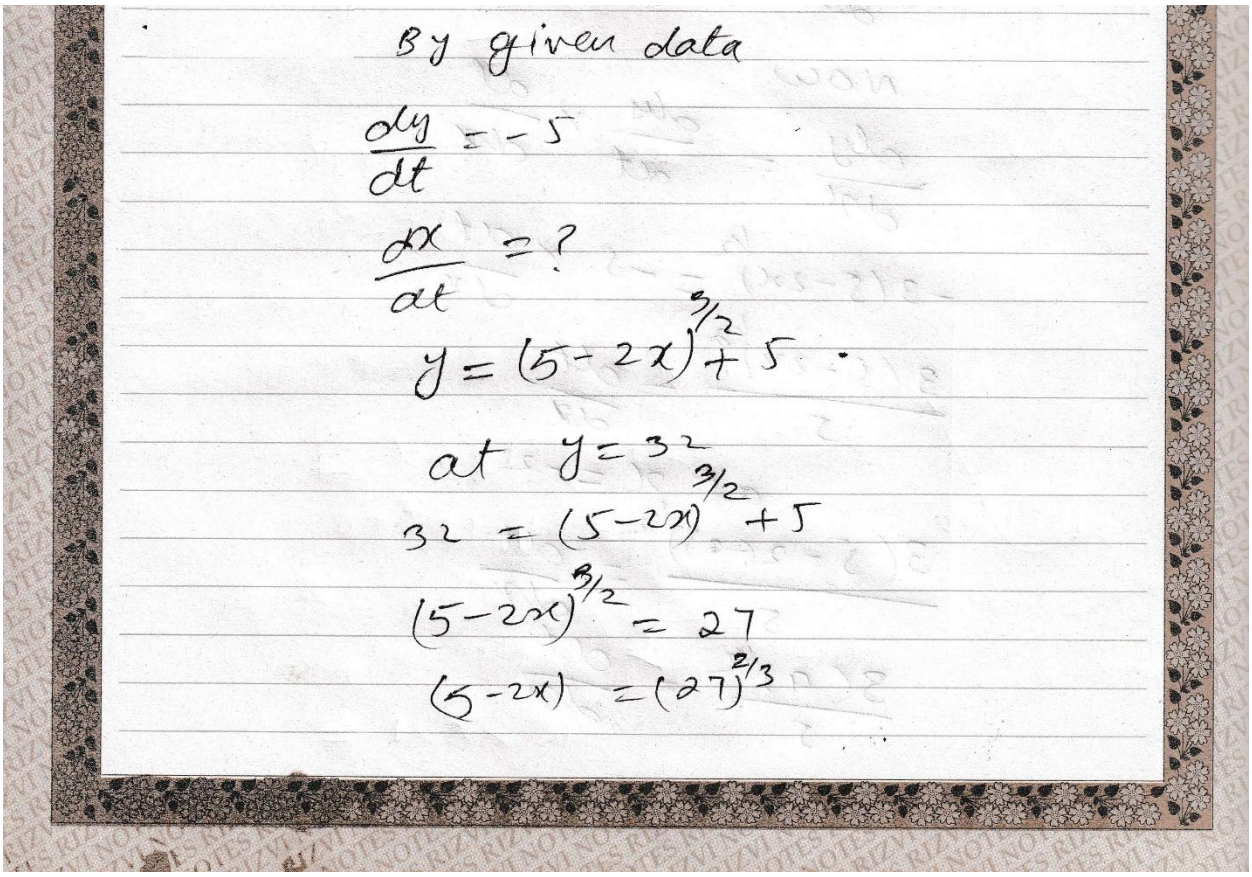
(b) Point A on the curve has y-coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3.

Find the equation of the perpendicular bisector of AB. Give your answer in the form

$ax + by + c = 0$, where a, b and c are integers.

Solution:

(a)



$$5-2x = (3^3)^{2/3}$$

$$5-2x = 3^2 = 9$$

$$2x = -4$$

$$x = -2$$

As $y = (5-2x)^{3/2} + 5$

Differentiate w.r.t x

$$\frac{dy}{dx} = \frac{3}{2}(5-2x)^{\frac{3}{2}-1}(0-2) + 0$$

$$\frac{dy}{dx} = -3(5-2x)^{1/2}$$

Now

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$-3(5-2x)^{1/2} = -5 \times \frac{dt}{dx}$$

$$\frac{3(5-2x)^{1/2}}{5} = \frac{dt}{dx}$$

at $x = -2$

$$\frac{3(5-2(-2))^{1/2}}{5} = \frac{dt}{dx}$$

$$\frac{3(9)^{1/2}}{5} = \frac{dt}{dx}$$

DATE :-

$$\frac{3\left(\frac{2}{3}\right)^{1/2}}{5} = \frac{dt}{dx}$$

$$\frac{9}{5} = \frac{dt}{dx}$$

$$\frac{dx}{dt} = \frac{5}{9}$$

(b)

point A on the curve has y-coordinate 32. The point B on the curve is such that the gradient of the curve at B is -3. Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by + c = 0$, where a, b, c are integers.

Solution

Equation of the curve

$$y = (5-2x)^{3/2} + 5 \rightarrow (1)$$

differentiate w.r.t x

$$\frac{dy}{dx} = 3/2(5-2x)^{3/2-1}(-2) + 0$$

$$\frac{dy}{dx} = -3(5-2x)^{1/2}$$

As at point B, $\frac{dy}{dx} = -3$

$$\Rightarrow -3 = -3(5-2x)^{1/2}$$

$$1 = (5-2x)^{1/2}$$

$$5-2x = 1$$

$$-2x = -4$$

$$x = 2$$

put $x = 2$ in equation (1), we have

$$y = (5-2(2))^{3/2} + 5$$

$$= (5-4)^{3/2} + 5$$

$$= 1 + 5$$

$$y = 6$$

$$B(2, 6)$$

As for point A, $y = 32$

put $y = 32$ in (1)

$$32 = (5-2x)^{3/2} + 5$$

$$27 = (5-2x)^{3/2}$$

$$(3^3)^{2/3} = (5-2x)$$

DATE :-

$$5 - 2x = 9$$

$$2x = 5 - 9$$

$$+2x = -4$$

$$x = -2$$

$$\Rightarrow A(-2, 32)$$

gradient of line AB

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 32}{2 - (-2)}$$

$$= -\frac{26}{4}$$

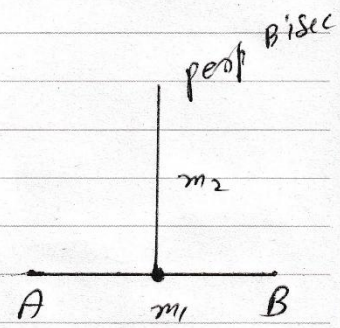
$$m_1 = -\frac{13}{2}$$

gradient of the perpendicular bisector

$$m_1 \times m_2 = -1$$

$$-\frac{13}{2} \times m_2 = -1$$

$$m_2 = \frac{2}{13}$$



As the mid point of AB lie on perpendicular bisector

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-2+2}{2}, \frac{32+6}{2}\right)$$

$$M = (0, 19)$$

Equation of the bisector

$$y - y_1 = m_2(x - x_1)$$

at $M(0,19)$ and $m_2 = \frac{2}{13}$, we have

$$y - 19 = \frac{2}{13}(x - 0)$$

$$13(y - 19) = 2x$$

$$13y - 247 = 2x$$

$$2x - 13y + 247 = 0$$