Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2024**

Question No (5)

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Question No (5)

The first and second terms of an arithmetic progression are $\tan\theta$ *and* $\sin\theta$ *respectively*, *where* $0 < \theta < \frac{\pi}{2}$.

(a) *Given that* $\theta = \frac{\pi}{4}$, *find the exact sum of the first* 40 *terms of the progression.*

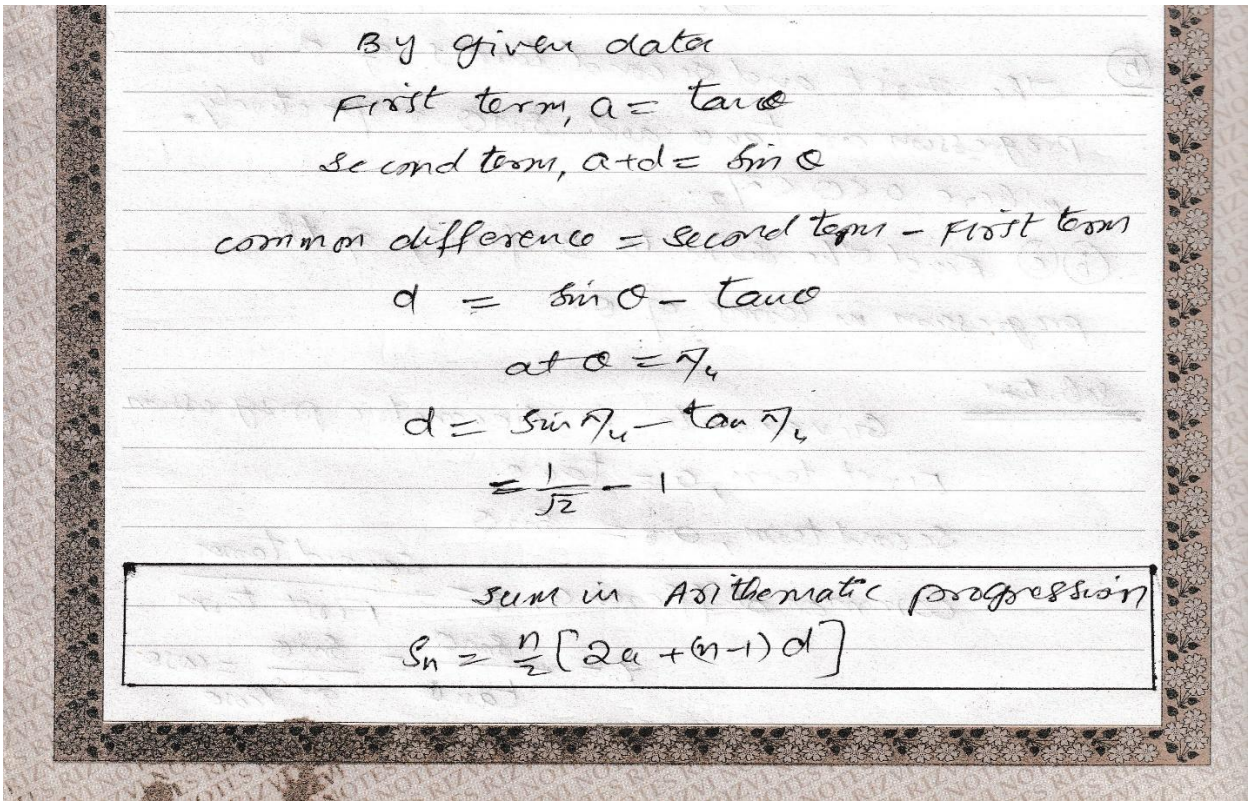
The first and second terms of a geometric progression are $\tan\theta$ *and* $\sin\theta$ *respectively*, *where* $0 < \theta < \frac{\pi}{2}$.

(b) (i) *Find the sum to infinity of the pogression in terms of* θ.

(ii) Given that $\theta = \frac{\pi}{3}$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures.

Solution:

(a)



$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{40} = \frac{40}{2}\left[2(\tan\theta) + (40-1)\left(\frac{1}{\sqrt{2}} - 1\right)\right]$$

$$= 20\left[2\tan\underset{\text{at } \theta = \pi/4}{\pi/4} + 39\left(\frac{1}{\sqrt{2}} - 1\right)\right]$$

$$= 20\left[2(1) + 39\left(\frac{1}{\sqrt{2}} - 1\right)\right]$$

$$= 40 + \frac{780}{\sqrt{2}} - 780$$

$$S_{40} = \frac{780}{\sqrt{2}} - 740$$

(b)(i)

: Given data in Geometric progression

First term, $a = \tan\theta$

Second term, $ar = \sin\theta$

common difference, $r = \dfrac{\text{second term}}{\text{first term}}$

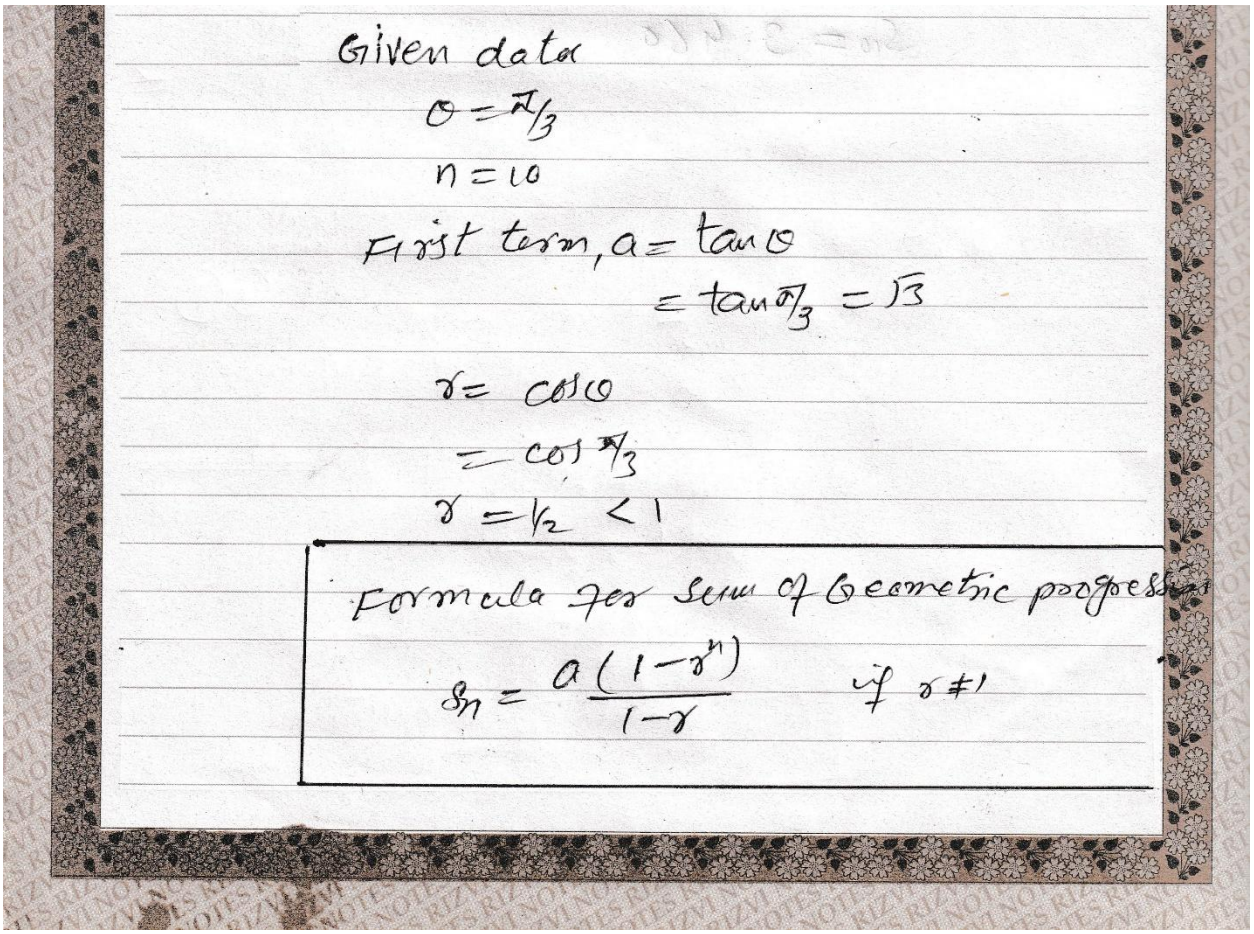
$$r = \frac{\sin\theta}{\tan\theta} = \frac{\sin\theta}{\sin\theta/\cos\theta} = \cos\theta$$

sum to infinity formula for G.P

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{\tan\theta}{1-\cos\theta}$$

(ii)



Given data

$\theta = \pi/3$

$n = 10$

First term, $a = \tan\theta$

$= \tan\pi/3 = \sqrt{3}$

$r = \cos\theta$

$= \cos\pi/3$

$r = 1/2 < 1$

Formula for sum of Geometric progression

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if } r \neq 1$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \sqrt{3}\,\frac{(1-(1/2)^{10})}{1-1/2}$$

$$= \frac{\sqrt{3}\,(1-(\frac{1}{2})^{10})}{1/2}$$

$$S_{10} = 2\sqrt{3}\,(1-(\tfrac{1}{2})^{10})$$

$$= 2\sqrt{3}\,(1-0.000976)$$

$$= 2\sqrt{3}\,(0.9990)$$

$$S_{10} = 3.460$$