

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

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Question No (7)

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Question No (7)

The equation of a circle is $(x - 6)^2 + (y + a)^2 = 18$. The line with equation $y = 2a - x$ is a tangent to the circle.

(a) Find the two possible values of the constant a .

(b) For the greater value of a , find the equation of the diameter which is perpendicular to the given tangent.

Solution:

(a)

Equation of circle
 $(x-6)^2 + (y+a)^2 = 18 \rightarrow (1)$

Equation of line
 $y = 2a - x \rightarrow (2)$

As line and circle intersect at tangent point. put value of (2) in (1)

$$(x-6)^2 + (2a-x+a)^2 = 18$$

$$(x-6)^2 + (3a-x)^2 = 18$$

$$x^2 - 12x + 36 + 9a^2 - 6ax + x^2 = 18$$

$$2x^2 - (12 + 6a)x + 36 + 9a^2 - 18 = 0$$

$$2x^2 - (12 + 6a)x + 18 + 9a^2 = 0 \rightarrow (3)$$

The diagram shows a circle and a line tangent to it at a single point. The circle is labeled 'circle' and the line is labeled 'line'.

As the line is tangent to the curve, so

$$\text{discriminant} = b^2 - 4ac = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$(12+6a)^2 - 4(2)(9a^2+18) = 0 \quad \text{y from (3)}$$

$$(12+6a)^2 - 8(9a^2+18) = 0$$

$$b = 12 + 6a$$

$$c = 9a^2 + 18$$

$$a = 2$$

$$(12)^2 + 144a + 36a^2 - 72a^2 - 144 = 0$$

$$144 + 144a + 36a^2 - 72a^2 - 144 = 0$$

$$144a - 36a^2 = 0$$

$$2(72a - 18a^2) = 0$$

$$72a - 18a^2 = 0$$

$$2(36a - 9a^2) = 0$$

$$36a - 9a^2 = 0$$

$$9(4a - a^2) = 0$$

$$4a - a^2 = 0$$

$$a(4-a) = 0$$

$$a = 0, \quad 4-a = 0$$

$$a = 0, \quad a = 4$$

(b)

Now at $a=4$, Equation of circle and line becomes.

$$(x-6)^2 + (y+4)^2 = 18 \rightarrow (1)$$

and line

$$y = 2(4) - x$$

$$y = 8 - x$$

$$y = -x + 8$$

$$m = -1$$

By comparing with
 $y = mx + c$

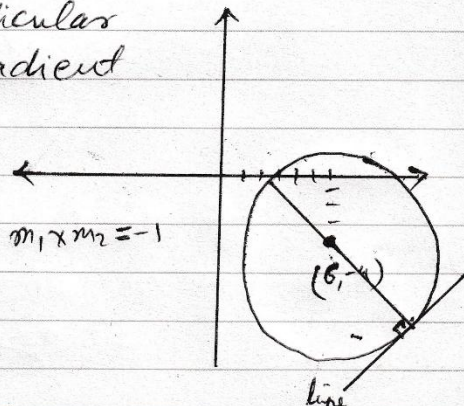
Centre of the circle from (1)
is $C(6, -4)$

As the diameter, perpendicular
to the tangent, so gradient
of diameter line is

$$= -\frac{1}{m}$$

$$= -\frac{1}{-1} = 1$$

$$m_1 \times m_2 = -1$$



Equation of line with gradient
and one point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

gradient of diameter = 1

point on parameter is, $(6, -4)$

Equation of diameter

$$y + 4 = 1(x - 6)$$

$$y = x - 6 - 4$$

$$y = x - 10$$

So the equation of the diameter is

$$y = x - 10$$