Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2024**

Question No(8)

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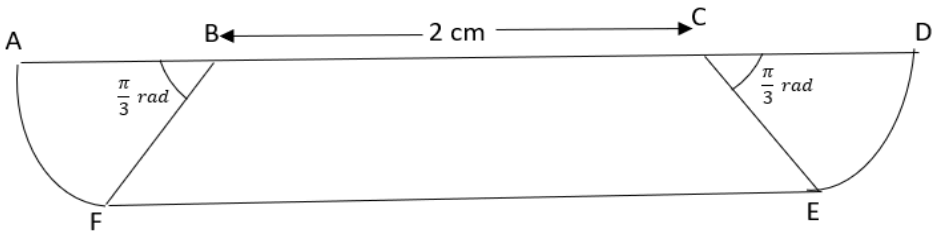
Question No (8)



The diagram shows a symmetrical plate ABCDEF. The line ABCD is straight and the length of BC is 2 cm. Each of the two sectors ABF and DCE is of radius r cm and each of the angle's ABF and DCE is equal to $\frac{\pi}{3}$ *radians.*

(a) It is given that $r = 0.4\ cm$.

(i) Show that the length EF=2.4 cm

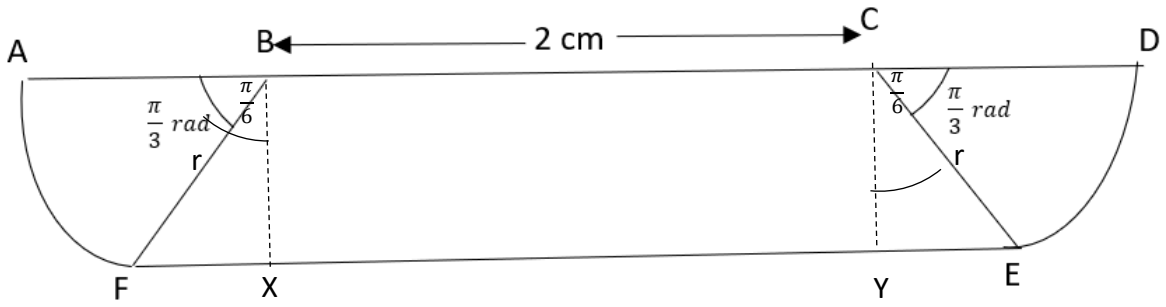
(ii) Find the area of the plate. Give your answer correct to 3 significant figures.

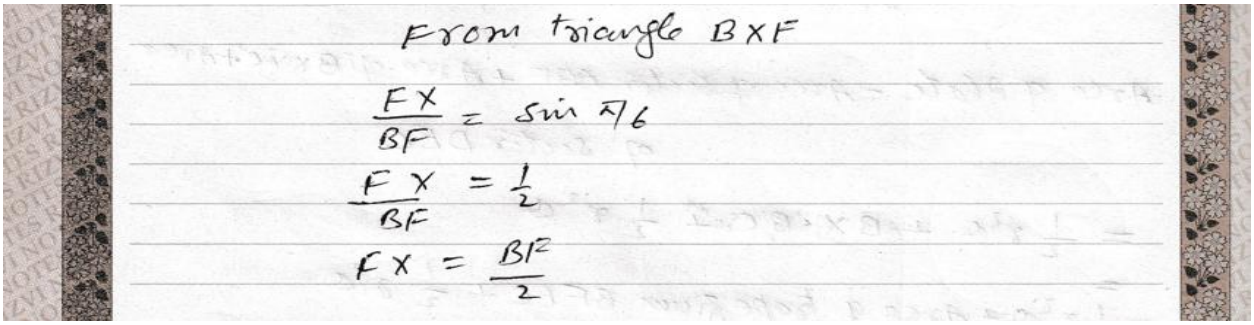
(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r. Give your answer correct to 3 significant figures.

Solution:

(a)(i)





$FX = \frac{0.4}{2}$ $\therefore BF = 0.4 = r$

$FX = 0.2$

Length of $EF = FX + XY + YE$ $\therefore XY = BC$

$= 0.2 + 2 + 0.2$ $FX = YE = 0.2$

$EF = 2.4$

(ii) Find the area of the plate. Give your answer correct to 3 significant figures.

Solution

From the figure of plate

$BX = \sqrt{(BF)^2 - (FX)^2}$

$= \sqrt{(0.4)^2 - (0.2)^2}$

$= \sqrt{0.16 - 0.04}$

$= \sqrt{0.12}$

$BX = 0.346$

Area of Plate = Area of sector ABF + Area of trapezium + Area of sector DEC

$= \frac{1}{2} r^2\theta$ + Area of trapezium + $\frac{1}{2} r^2\theta$

$= \frac{1}{2} r^2\theta$ + Area of trapezium BFEC + $\frac{1}{2} r^2\theta$

$= \frac{1}{2}(0.4)^2\frac{\pi}{3} + \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height} + \frac{1}{2}(0.4)^2\frac{\pi}{3}$

$= 0.0837 + \frac{1}{2} \times (|BC| + |FE|) \times |BX| + 0.0837$

$= 0.0837 + \frac{1}{2} \times (2 + 2.4) \times 0.346 + 0.0837$

$= 0.930$

(b) It is given instead that the perimeter of the plate is 6 cm. Find the value of r. Give your answer correct to 3 significant figures.

Solution

perimeter of plate

$= AB + BC + CD + \overset{\frown}{DE} + EF + \overset{\frown}{AF}$

$= r + 2 + r + r(\pi/3) + 2.4 + r(\pi/3)$

$s = r\theta$

$\overset{\frown}{AF} = \overset{\frown}{DE} = r\theta$

As perimeter of plate is 6

$\Rightarrow 6 = r + 2 + r + \frac{\pi r}{3} + 2.4 + \frac{\pi r}{3}$

$6 - 2 - 2.4 = 2r + \frac{2\pi r}{3}$

$1.6 = \left(2 + \frac{2\pi}{3}\right) r$

$r = \frac{1.6}{\left(2 + \frac{2\pi}{3}\right)}$

$r = \frac{1.6}{2 + 2.094}$

$= \frac{1.6}{4.094}$

$r = 0.391$