Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2024**

Question No (9)

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Question No (9)

A function f *such that* $f'(x) = 6(2x-3)^2 - 6x$ *for* $x \in R$

(a) Determine the set of values of x for which $f(x)$ is decreasing.

(b) Given that $f(1) = -1$, *find* $f(x)$.

Solution:

(a)

As $f(x)$ is decreasing, so

$f'(x) < 0$

$6(2x-3)^2 - 6x < 0$

$6(4x^2 - 12x + 9) - 6x < 0$

$24x^2 - 72x + 54 - 6x < 0$

$24x^2 - 78x + 54 < 0$

$6(4x^2 - 13x + 9) < 0$

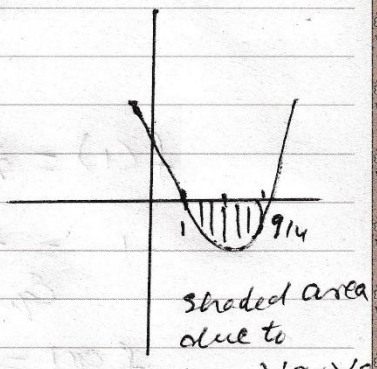
$\Rightarrow 4x^2 - 13x + 9 < 0$

$4x^2 - 9x - 4x + 9 < 0$

$x(4x-9) - 1(4x-9) < 0$

$(4x-9)(x-1) < 0$

$\Rightarrow 1 < x < 9/4$



(b)

(b) given that $f'(1) = -1$ find $f(x)$

Solution

$$f'(x) = 6(2x-3)^2 - 6x$$

$$\frac{df}{dx} = 6(2x-3)^2 - 6x$$

$$df = [6(2x-3)^2 - 6x]\,dx$$

integrate

$$f = \int [6(2x-3)^2 - 6x]\,dx$$

$$f(x) = \frac{6}{2}\frac{(2x-3)^{2+1}}{2+1} - \frac{6x^{1+1}}{1+1} + c$$

$$f(x) = (2x-3)^3 - 3x^2 + c \longrightarrow (1)$$

c is constant of integration,

given condition

$$f(1) = -1$$

Equation (1) becomes

$$f(1) = (2(1)-3)^3 - 3(1)^2 + c$$

$$-1 = -1 - 3 + c \Rightarrow c = 3$$

Equation (1) becomes

$$f(x) = (2x-3)^3 - 3x^2 + 3$$