Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2024**

Question No (10)

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Question No (10)

$(\boldsymbol{a})$ *By writing* $y = sec^3\theta$ *as* $\frac{1}{cos^3\theta}$ *, show that* $\frac{dy}{d\theta} = 3\sin\theta\ sec^4\theta.$

$(\boldsymbol{b})$ *The variables* x *and* θ *satisfy the differential equation*

$$(\boldsymbol{x^2 + 9})\ \boldsymbol{\sin\theta}\ \frac{d\theta}{dx} = (\boldsymbol{x + 3})\ cos^4\theta.$$

it is given that $x = 3$ *when* $\theta = \frac{\pi}{3}$.

Solve the differential eqaution to find the value of $\cos\theta$ *when* $x = 0$.

Give your answer correct to 3 significant figures.

Solution:(a)

$y = \sec^3\theta$

$y = \frac{1}{\cos^3\theta}$

$y = \cos^{-3}\theta$

Differentiate w.r.t θ

$\frac{dy}{d\theta} = -3\cos^{-3-1}\theta\,(-\sin\theta)$

$= -3\cos^{-4}\theta\,(-\sin\theta)$

$= 3\cos^{-4}\theta\sin\theta$

$= \frac{3\sin\theta}{\cos^4\theta}$

$\frac{dy}{d\theta} = 3\sin\theta\sec^4\theta$

DATE :-

(b) The variables x and θ satisfy the differential equation

$$(x^2+9)\sin\theta \frac{d\theta}{dx} = (x+3)\cos^4\theta$$

It is given that $x=3$ when $\theta=\pi/3$. solve the differential equation to find the value of $\cos\theta$ when $x=0$. Give your answer correct to 3 significant figures

Solution

$$(x^2+9)\sin\theta \frac{d\theta}{dx} = (x+3)\cos^4\theta$$

separate variables

$$\frac{\sin\theta}{\cos^4\theta}\, d\theta = \left(\frac{x+3}{x^2+9}\right) dx$$

$$\sin\theta \cos^{-4}\theta\, d\theta = \frac{x}{x^2+9}\, dx + \frac{3}{x^2+9}\, dx$$

$$-\int(-\sin\theta \cos^{-4}\theta\, d\theta) = \frac{1}{2}\int \frac{2x}{x^2+9}\, dx + 3\int \frac{dx}{x^2+9}$$

$$\frac{\cos^{-4+1}}{-4+1} = \frac{1}{2}\ln(x^2+9) + 3\cdot\frac{1}{3}\tan^{-1}(x/3) + c$$

$$-\frac{\cos^{-3}\theta}{-3} = \frac{1}{2}\ln(x^2+9) + \tan^{-1}(x/3) + c$$

$$\frac{1}{3\cos^3\theta} = \frac{1}{2}\ln(x^2+9) + \tan^{-1}\left(\frac{x}{3}\right) + c$$

$$\sec^3\theta \frac{1}{\sec^3\theta} = \frac{3}{2}\ln(x^2+9) + 3\tan^{-1}\left(\frac{x}{3}\right) + K \rightarrow (1)$$

at $x=3$, $\theta = \pi/3$ given

$$\left(\frac{1}{\cos\pi/3}\right)^3 = \frac{3}{2}\ln(3)^2+9) + 3\tan^{-1}(3/3) + K$$

$$\left(\frac{1}{1/2}\right)^3 = \frac{3}{2}\ln(9+9) + 3\tan^{-1}(1) + K$$

$$(2)^3 = \frac{3}{2}\ln(18) + 3\pi/4 + K$$

$$8 = \frac{3}{2}\ln 18 + \frac{3\pi}{4} + K$$

$$K = 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}$$

put value of K in (1)

$$\sec^3\theta = \frac{3}{2}\ln(x^2+9) + 3\tan^{-1}\left(\frac{x}{3}\right) + 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}$$

at $x=0$

$$\sec^3\theta = \frac{3}{2}\ln 9 + 0 + 8 - \frac{3}{2}\ln 18 - \frac{3\pi}{4}$$

$$\sec^3\theta = 3.2958 + 8 - 4.335 - 2.3561 = 4.6042$$

$$\cos^3\theta = \frac{1}{4.6042} = 0.2171$$

$$\cos^3\theta = 0.2171 \quad 0.3333$$

$$\cos\theta = (0.271) = 0.601$$

$$\Rightarrow \cos\theta = 0.601$$