

Cambridge International AS & A Level

Mathematics

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Paper 3 Pure Mathematics 3

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Question No (3)

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Question No (3)

The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line.

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form.

Solution:

(a)

$$\begin{aligned}
 a^{2y-1} &= b^{x-y} \\
 \ln(a^{2y-1}) &= \ln(b^{x-y}) \\
 (2y-1) \ln a &= (x-y) \ln b \\
 2y \ln a - \ln a &= x \ln b - y \ln b \\
 2y \ln a + y \ln b &= x \ln b + \ln a \\
 y(2 \ln a + \ln b) &= \ln b x + \ln a \\
 y(\ln a^2 + \ln b) &= \ln b x + \ln a \\
 y(\ln(a^2 b)) &= \ln b x + \ln a \\
 y &= \frac{\ln b}{\ln a^2 b} x + \frac{\ln a}{\ln a^2 b} \\
 &\text{compare it with} \\
 y &= mx + c
 \end{aligned}$$

so its graph is a straight line due to one power of x and y .

⑥ Given that $a = b^3$, state the equation of the straight line in the form, $y = px + q$, where p and q are rational numbers in their simplest form.

Solution

given equation

$$a^{2y-1} = b^{x-y} \rightarrow \textcircled{1}$$

given condition

$$a = b^3$$

Equation $\textcircled{1}$ becomes

$$(b^3)^{2y-1} = b^{x-y}$$

$$b^{6y-3} = b^{x-y}$$

$$\ln(b^{6y-3}) = \ln(b^{x-y})$$

$$6y-3 \ln b = (x-y) \ln b$$

$$6y-3 = x-y$$

$$6y+y = x+3$$

$$7y = x+3$$

$$y = \frac{1}{7}x + \frac{3}{7}$$

