

Cambridge International AS & A Level

Mathematics

9709/32

Paper 3 Pure Mathematics 3

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Question No (4)

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Question No (4)

The equation of a curve is $y e^{2x} + y^2 e^x = 6$.

Find the gradient of the curve at the point where $y = 1$.

Solution:

$$\begin{aligned}
 & y e^{2x} + y^2 e^x = 6 \rightarrow (1) \\
 & \text{at } y=1 \text{ (given)} \\
 & (1) e^{2x} + 1^2 e^x = 6 \\
 & e^{2x} + e^x - 6 = 0 \\
 & \text{put } e^x = t \\
 & t^2 + t - 6 = 0 \\
 & \text{factorize} \\
 & t^2 + 3t - 2t - 6 = 0 \\
 & t(t+3) - 2(t+3) = 0 \\
 & (t+3)(t-2) = 0 \\
 & \begin{array}{l|l}
 t+3=0 & t-2=0 \\
 t=-3 & t=2 \\
 e^x=-3 & e^x=2 \\
 \ln e^x = \ln(-3) & \\
 x \ln e = \ln(-3) &
 \end{array}
 \end{aligned}$$

$$x = \ln(-3)$$

does not exist

$$e^x = 2$$

Now from ①

$$y e^{2x} + y^2 e^x = 6$$

differentiate w.r.t x

$$\left[y \frac{d}{dx}(e^{2x}) + e^{2x} \frac{dy}{dx} \right] + \left[y^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(y^2) \right] = 0$$

$$y(e^{2x} \cdot 2) + e^{2x} \frac{dy}{dx} + y^2 e^x + e^x (2y \frac{dy}{dx}) = 0$$

$$[e^{2x} + 2y e^x] \frac{dy}{dx} = -2y e^{2x} - y^2 e^x$$

$$\frac{dy}{dx} = \frac{-2y e^{2x} - y^2 e^x}{e^{2x} + 2y e^x}$$

at $y=1, e^x=2$

$$\frac{dy}{dx} = \frac{-2(1)(2)^2 - (1)^2 2}{(2)^2 + 2(1)2}$$

$$\frac{dy}{dx} = \frac{-8-2}{8} = \frac{-10}{8} = -\frac{5}{4}$$

