

Cambridge International AS & A Level

Mathematics

9709/32

Paper 3 Pure Mathematics 3

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Question No (5)

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Question No (5)

(a) It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$.

(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

converges then it converges to the root of the equation in part (a).

(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

Solution:

(a)

$$\begin{aligned}
 e^{2x} &= 5 + \cos 3x \\
 f(x) &= 5 + \cos 3x - e^{2x} \\
 f(0.72) &= 5 + \cos 3(0.72) - e^{2(0.72)} \\
 &= 5 + 0.7190 - 5.3201 \\
 f(0.72) &= 0.3989 > 1 \\
 \text{now} \\
 f(0.79) &= 5 + \cos 3(0.79) - e^{2(0.79)} \\
 &= 5 + 0.7889 - 5.8374 \\
 &= -0.0485 < 1
 \end{aligned}$$

As the sign of $f(0.72)$ and $f(0.79)$ changed, so one root lies in the interval $0.7 < x < 0.8$.

⑥ Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

converges then it converges to the root of the equation in part (a)

Solution

given formula

$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

removing the subscripts, we have

$$x = \frac{1}{2} \ln(5 + \cos 3x)$$

$$2x = \ln(5 + \cos 3x)$$

taking the power e

$$\frac{2x}{e} = \ln(5 + \cos 3x)$$

$$e^x = 5 + \cos 3x$$

it's same as of part (a), so it converges to the root of the equation.

DATE:-

© use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

solution

$$x_{n+1} = \frac{1}{2} \ln(5 + \cos 3x_n)$$

using initial value

$$x_0 = \frac{a+b}{2}$$

$$= \frac{0.7 + 0.8}{2}$$

$$x_0 = 0.75$$

At $n=0$

$$x_1 = \frac{1}{2} \ln(5 + \cos 3x_0)$$

$$x_1 = \frac{1}{2} \ln(5 + \cos 3(0.75)) \quad \because x_0 = 0.75$$

$$x_1 = 0.73759$$

(make sure you have changed the mod to radians)

At $n=1$

$$x_2 = \frac{1}{2} \ln(5 + \cos 3x_1)$$

$$= \frac{1}{2} \ln(5 + \cos 3(0.73759))$$

$$= \frac{1}{2} \ln(5 + \cos 2.21277)$$

$$x_2 = 0.74094$$

At $n=2$

$$x_3 = \frac{1}{2} \ln(5 + \cos 3x_2)$$

$$= \frac{1}{2} \ln(5 + \cos 3(0.74094)) \quad \because x_2 = 0.74094$$

$$x_3 = \frac{1}{2} \ln(5 + \cos 2.22282)$$

$$x_3 = 0.74002$$

So, The root start repeating, The root upto 3 decimal places is

$$x = 0.740$$