

Cambridge International AS & A Level

Mathematics

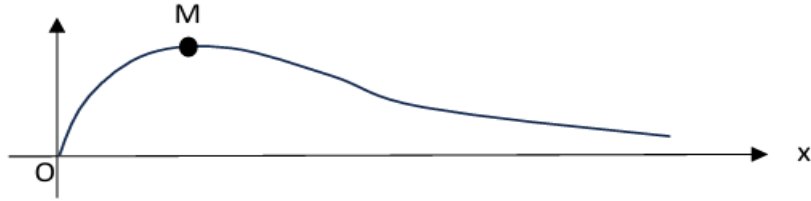
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Paper 3 Pure Mathematics 3

May/June 2024

Question No (6)

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Question No (6).

The diagram shows the curve $y = x e^{-ax}$, where a is a positive constant, and its maximum point M .

(a) Find the exact coordinates of M .

(b) Find the exact value of $\int_0^{\frac{2}{a}} x e^{-ax} dx$.

Solution:

(a)

$$y = x e^{-ax} \rightarrow \textcircled{1}$$

Differentiate w.r.t x

$$\frac{dy}{dx} = x \frac{d}{dx}(e^{-ax}) + e^{-ax} \frac{d}{dx}(x)$$

Formula

$$\boxed{\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}}$$

$$\frac{dy}{dx} = x(e^{-ax}(-a)) + e^{-ax}(1)$$

$$= -ax e^{-ax} + e^{-ax}$$

$$\frac{dy}{dx} = e^{-ax} [-ax + 1]$$

At the maximum point M, $\frac{dy}{dx} = 0$

$$\Rightarrow e^{-ax} (-ax + 1) = 0$$

$$\Rightarrow e^{-ax} \neq 0$$

$$-ax + 1 = 0$$

$$x = \frac{1}{a}$$

put $x = \frac{1}{a}$ in ①

$$y = \frac{1}{a} e^{-a(\frac{1}{a})}$$

$$= \frac{1}{a} e^{-1}$$

$$y = \frac{1}{ae}$$

∴ The maximum point M $(\frac{1}{a}, \frac{1}{ae})$.

⑥ Find the exact value of $\int_0^{\frac{2}{a}} x e^{-ax} dx$

Solution

$$\int_0^{\frac{2}{a}} x e^{-ax} dx$$

DATE:-

Formula for integration by parts

$$\int u v dx = u \int v dx - \int \left(\frac{d}{dx} u \right) \left(\int v dx \right) dx$$

$$= x \int e^{-ax} dx - \int \left(\frac{d}{dx} (x) \right) \left(\int e^{-ax} dx \right)$$

$$= x \frac{e^{-ax}}{-a} - \int (1) \frac{e^{-ax}}{-a} dx$$

$$= -\frac{x}{a} e^{-ax} + \frac{1}{a} \int e^{-ax} dx$$

$$= \left[-\frac{x}{a} e^{-ax} + \frac{1}{a} \frac{e^{-ax}}{-a} \right]_0^{2/a}$$

$$= -\frac{1}{a} \left(\frac{2}{a} \right) e^{-a(\frac{2}{a})} + \frac{1}{a} \left(\frac{1}{-a} \right) e^{-a(\frac{2}{a})} - \left[0 + \frac{1}{a} \left(\frac{1}{-a} \right) e^0 \right]$$

$$= -\frac{2}{a^2} e^{-2} - \frac{1}{a^2} e^{-2} + \frac{1}{a^2}$$

$$= \frac{1}{a^2} (1 - 3e^{-2})$$

