Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 May/June 2024

Question No (7)

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Question No (7)

(a) Show that $cos^4\theta - sin^4\theta \equiv \cos 2\theta$.

(b) Hence find the exact value of $\int_{\frac{\pi}{8}}^{\frac{\pi}{8}}(cos^4\theta - sin^4\theta + 4\,sin^2\,\theta\,cos^2\theta)\,d\theta$.

Solution:

(a)

$\cos^4\theta - \sin^4\theta = \cos 2\theta$

L.H.S

$\cos^4\theta - \sin^4\theta$

$(\cos^2\theta)^2 - (\sin^2\theta)^2$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$ $\quad \because a^2 - b^2 = (a-b)(a+b)$

$(\cos^2\theta - \sin^2\theta)$ $\quad \cos^2\theta + \sin^2\theta = 1$

$\cos(\theta + \theta)$

$\cos 2\theta$ $\quad \because \cos 2\theta = \cos^2\theta - \sin^2\theta$

L.H.S = R.H.S

Trigonometry Formula

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

DATE :-

(b) Hence find the exact value of

$$\int_{-\pi/8}^{\pi/8} (\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta)\, d\theta$$

<u>Solution</u>

$$= \int_{-\pi/8}^{\pi/8} (\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta)\, d\theta$$

$$= \int_{-\pi/8}^{\pi/8} (\cos 2\theta + 4\sin^2\theta\cos^2\theta)\, d\theta$$

From part (a) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$$= \int_{-\pi/8}^{\pi/8} (\cos 2\theta + (2\sin\theta\cos\theta)^2)\, d\theta$$

$$= \int_{-\pi/8}^{\pi/8} (\cos 2\theta + (\sin 2\theta)^2)\, d\theta$$

$\because \sin 2\theta = 2\sin\theta\cos\theta$

$$= \int_{-\pi/8}^{\pi/8} (\cos 2\theta + \sin^2 2\theta)\, d\theta$$

$$= \int_{-\pi/8}^{\pi/8} \left[\cos 2\theta + \left(\frac{1 - \cos 4\theta}{2}\right)\right] d\theta$$

$$= \int_{-\pi/8}^{+\pi/8} \cos 2\theta \, d\theta + \int_{-\pi/8}^{\pi/8} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$$

$$= \int_{-\pi/8}^{\pi/8} \cos 2\theta \, d\theta + \frac{1}{2} \int_{-\pi/8}^{\pi/8} (1-\cos 4\theta) \, d\theta$$

$$= \left|\frac{\sin 2\theta}{2}\right|_{-\pi/8}^{\pi/8} + \frac{1}{2}\left|\left(\theta - \frac{\sin 4\theta}{4}\right)\right|_{-\pi/8}^{\pi/8}$$

$$= \frac{1}{2}\left(\sin \pi/4 - \sin(-\pi/4)\right) + \frac{1}{2}\left[\left(\pi/8 - \frac{1}{4}\sin \pi/2\right) - \left(-\pi/8 - \frac{\sin(-\pi/2)}{4}\right)\right]$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left[\frac{\pi}{8} - \frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right]$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right) + \frac{1}{2}\left[\frac{2\pi}{8} - \frac{1}{2}\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{\pi}{8} - \frac{1}{4}$$

$$= \frac{2}{2} \times \frac{1}{\sqrt{2}} + \frac{\pi}{8} - \frac{1}{4}$$

$$= \frac{\sqrt{2}}{2} + \frac{\pi}{8} - \frac{1}{4}$$

$$\int_{-\pi/8}^{\pi/8} \left(\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\right) d\theta = \frac{\sqrt{2}}{2} + \frac{\pi}{8} - \frac{1}{4}$$