Cambridge International AS & A Level

Mathematics 9709/42

Paper 4 Mechanics May/June 2024

Question No(5)

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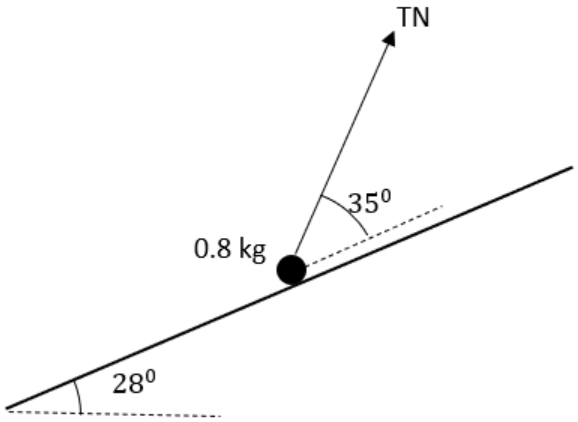
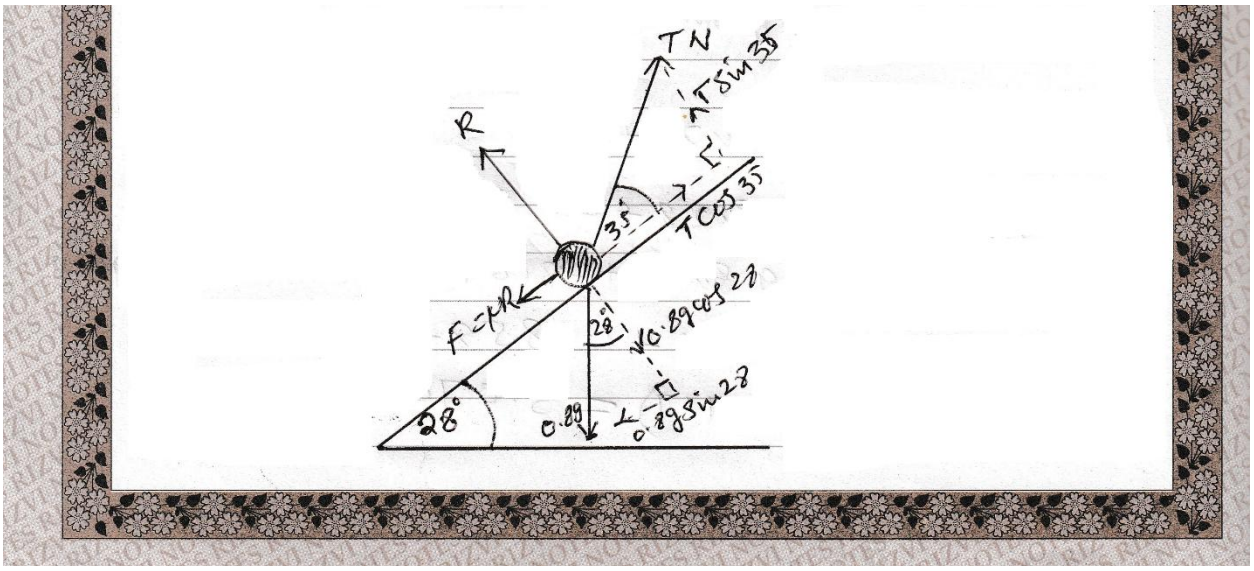


A particle of mass 0.8 kg lies on a rough plane which is inclined at an angle of 28° to the horizontal. The particle is kept in equilibrium by a force of magnitude T N. This force acts at an angle of 35° above a line of greatest slope of the plane (see diagram). The coefficient of friction between the particle and the plane is 0.2.

Find the least and greatest possible values of T.

Solution:



Resolve the forces $T\overline{N}$ and $0.8g$ into its rectangular components. as shown in the diagram.

Resolve the forces perpendicular to the inclined plane.

upward forces = downward forces.

$$R + T\sin 35 = 0.8g\cos 28$$

$$R = 0.8g\cos 28 - T\sin 35$$

$$R = 6.9294 - 0.5735T \rightarrow (1)$$

Case 1

T is greatest when friction adds to the opposing forces.

Resolve the forces along the inclined plane, we have from the diagram.

$T\cos 35 = 0.8g\sin 28 + F$ (Friction direction along the forces)

$T\cos 35 = 0.8g\sin 28 + \mu R$ $\because F = \mu R$

$T\cos 35 = 0.8g\sin 28 + 0.2R$ $\because \mu = 0.2$

$(0.8192)T = 0.8g\sin 28 + 0.2(6.9294 - 0.5735T)$ By (1)

$(0.8192)T = 3.6844 + 1.3858 - 0.1147T$

$T(0.8192 + 0.1147) = 5.0702$

$$T = \frac{5.0702}{0.9339}$$

$$T = 5.42$$

$$T = 5.4$$

Case 2

T is least when friction opposes the motion.

Resolve the forces along the inclined plane. Now in this case direction of friction will be opposite

$$T\cos 35 = 0.8g \sin 28 - F$$

$$T\cos 35 = 0.8g \sin 28 - \mu R$$

$$T\cos 35 = 0.8g \sin 28 - 0.2R$$

$$T\cos 35 = 0.8g \sin 28 - 0.2(6.9294 - 0.5735T)$$ from (1)

$$T(0.8191) = 3.6844 - 1.3858 + 0.1147T$$

$$T(0.8191 - 0.1147) = 2.2986$$

$$T = \frac{2.2986}{0.7044}$$

$$T = 3.3$$