

Cambridge International AS & A Level

Mathematics

9709/52

Paper 5 Probability & Statistics 1

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Question No (6)

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Question No (6)

The residents of Mahjing were asked to classify their local bus service:

- 25% of residents classified their service as good.
- 60% of residents classified their service as satisfactory.
- 15% of residents classified their service as poor.

(a) A random sample of 110 residents of Mahjing is chosen.

Use a suitable approximation to find the probability that fewer than 22 residents classified their bus service as good.

(b) For a random sample of 10 residents of Mahjing, find the probability that fewer than 8 classified their bus service as good or satisfactory.

(c) Three residents of Mahjing are selected at random.

Find the probability that one resident classified the bus service as good, one as satisfactory and one as poor.

Solution:

(a)

let X be the variable of binomial distribution

$$X \sim \text{Bin}(110, 0.25) \quad \text{or} \quad X \sim \text{Bin}(n, p)$$

$$n = 110$$

$$\text{As } np = 110 \times 0.25 \\ = 27.5 > 5$$

$$p = 25\%$$

$$= 0.25$$

$$\text{and } nq = 110 \times (1 - 0.25) \\ = 82.5 > 5$$

$$q = 1 - p$$

$$= 0.75$$

As $np > 5$ and $nq > 5$, so we can use normal approximation with continuity correction.

$$\Rightarrow \mu = np \\ = 110 \times 0.25 \\ = 27.5$$

$$\sigma = \sqrt{npq} \\ = \sqrt{110 \times 0.25 \times 0.75}$$

$$\sigma = 4.5415$$

$$P(X < 22) = P\left(Z < \frac{21.5 - 27.5}{4.5415}\right) \quad \text{By using continuity correction}$$

$$P(Z < -1.3212) = 1 - \Phi(1.3212)$$

$$= 1 - 0.9068$$

By normal distribution table

$$P(X < 22) = P(Z < -1.3212) = 0.0932$$

(b)

on the question statement

$$\begin{aligned}\text{good \%} &= 25\% \\ &= 0.25\end{aligned}$$

$$\begin{aligned}\text{satisfactory \%} &= 60\% \\ &= 0.60\end{aligned}$$

$$\text{poor \%} = 15\% = \frac{15}{100} = 0.15$$

or
good and satisfactory probability

$$p = 0.25 + 0.60$$

$$p = 0.85$$

let x be the number of residents out of 10
say good or satisfactory

$$X \sim \text{Bin}(n=10, p=0.85)$$

By given condition

$$P(X < 8) = P(X \leq 7)$$

$$\begin{aligned}&= \sum_{k=0}^7 \binom{10}{k} p^k q^{10-k} \\&= \binom{10}{0} (0.85)^0 (0.15)^{10} + \binom{10}{1} (0.85)^1 (0.15)^9 + \binom{10}{2} (0.85)^2 (0.15)^8 \\&\quad + \binom{10}{3} (0.85)^3 (0.15)^7 + \binom{10}{4} (0.85)^4 (0.15)^6 + \binom{10}{5} (0.85)^5 (0.15)^5 \\&\quad + \binom{10}{6} (0.85)^6 (0.15)^4 + \binom{10}{7} (0.85)^7 (0.15)^3\end{aligned}$$

$$P(X < 8) = 0.180$$

(c)

probability of each classification

$$\text{good} = 25\% = 0.25$$

$$\text{satisfactory} = 60\% = 0.60$$

$$\text{poor} = 15\% = 0.15$$

According to requirement

1 → good

1 → satisfactory

1 → poor

probability is

$$0.25 \times 0.60 \times 0.15$$

$$= 0.0225$$

As there are 3! for good, satisfactory and poor, so

$$P(1 \text{ good}, 1 \text{ satisfactory}, 1 \text{ poor}) = 6 \times 0.0225$$

$$= 0.135$$