Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2025**

Question No (8)

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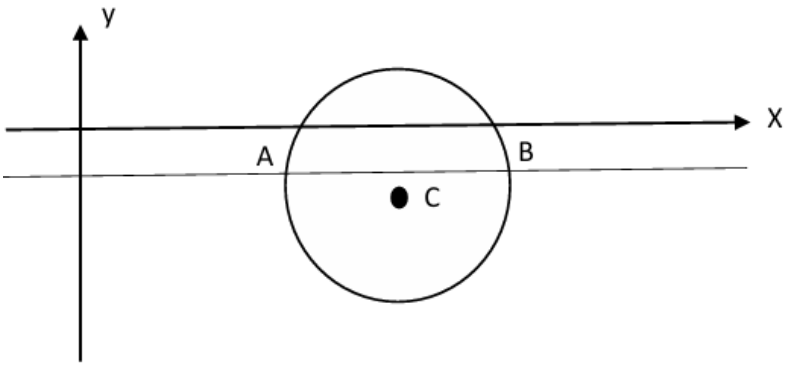
Question No (8)



The diagram shows the circle with $x^2 + y^2 - 14x + 8y + 36 = 0$ *and the line* $y = -2$.The line intersects the circle at the points A and B. The centre of the circle is C.

(a) Find the coordinates of A, B and C.

(b) Find the angle ACB in radians. Give your answer correct to 3 significant figures.

(c) The chord AB divides the circle into two segments.

Find the area of the larger segment.

Solution:

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Given equation of circle

$x^2+y^2-14x+8y+36=0 \longrightarrow (1)$

and line $y=-2 \longrightarrow (2)$

(a)

put $y=-2$ in Equation (1)

$x^2+(-2)^2-14x+8(-2)+36=0$

$x^2+4-14x-16+36=0$

$x^2-14x+24=0$

Factorize

$x^2-12x-2x+24=0$

$x(x-12)-2(x-12)=0$

$(x-12)(x-2)=0$

$x-12=0, \quad x-2=0$

$x=12, \quad x=2$

So $A(2,-2)$ and $B(12,-2)$

NOW we find centre of circle.

From (1)

$x^2+y^2-14x+8y+36=0$

$x^2-14x+y^2+8y=-36$

$x^2-2\times7x+y^2+2\times4y=-36$

$x^2 - 2\times 7x + (7)^2 + y^2 + 2\times 4y + (4)^2 = -36 + (7)^2 + (4)^2$

$(x-7)^2 + (y+4)^2 = -36 + 49 + 16$

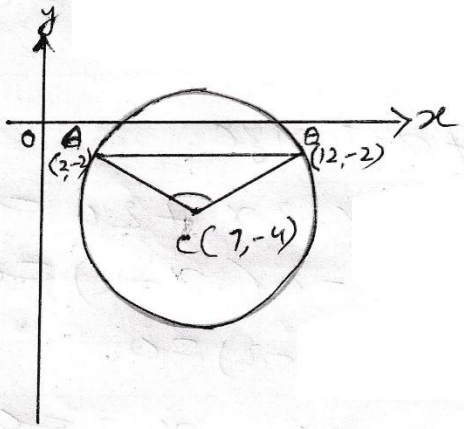
$(x-7)^2 + (y+4)^2 = 29$

$(x-7)^2 + (y+4)^2 = (\sqrt{29})^2$

$C(7,-4)$

$\therefore (x-a)^2 + (y-b)^2 = r^2$ is standard eqn of circle.

(b)



Distance formula

$A(x_1, y_1)$, $B(x_2, y_2)$

$$|AB| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$|AB| = \sqrt{(12-2)^2 + (-2+2)^2}$

$|AB| = \sqrt{(10)^2 + (0)^2} = 10$

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$$|AC| = \sqrt{(7-2)^2 + (-4+2)^2}$$

$$= \sqrt{(5)^2 + (-2)^2}$$

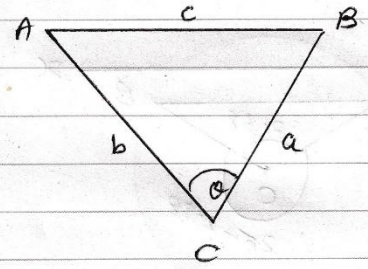
$$|AC| = \sqrt{25+4} = \sqrt{29}$$

$$|BC| = \sqrt{(7-12)^2 + (-2+4)^2}$$

$$= \sqrt{(5)^2 + (2)^2}$$

$$|BC| = \sqrt{25+4} = \sqrt{29}$$

Cosine law



$$c^2 = a^2 + b^2 - 2ab\cos C$$

By using the cosine law for triangle ACB, we have.

$(AB)^2 = (AC)^2 + (BC)^2 - 2\,|AC||BC|\cos A\hat{C}B$

$(10)^2 = (\sqrt{29})^2 + (\sqrt{29})^2 - 2\sqrt{29}\sqrt{29}\cos A\hat{C}B$

$100 = 29 + 29 - 2(29)\cos A\hat{C}B$

$58\cos A\hat{C}B = -100 + 58$

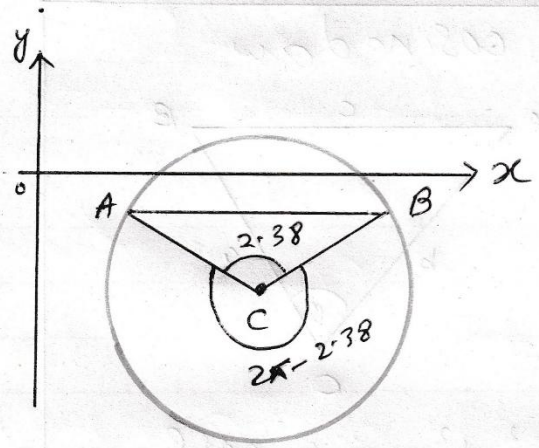
$\cos A\hat{C}B = \frac{-42}{58}$

$\cos A\hat{C}B = -0.7241$

$A\hat{C}B = \cos^{-1}(-0.7241)$ angle in radian

$A\hat{C}B = 2.38$ rad

(c)



Area of sector

$A = \frac{1}{2}r^2\theta$

Area of large sector $= \frac{1}{2}(|AC|)^2(2\pi - 2.38)$

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$= \frac{1}{2}(\sqrt{29})^2 (2\pi - 2.38)$

$= \frac{1}{2} \times 29 \times 3.903$

$= 56.587$

Area of triangle $ACB = \frac{1}{2} |AC| |BC| \sin \hat{C}$

$= \frac{1}{2} \times \sqrt{29} \times \sqrt{29} \sin 2.38$

$= 10$

Area of larger segment = Area of large sector + Area of triangle ACB

$= 56.587 + 10$

$= 66.58$

$= 66.6$