Cambridge International AS & A Level

Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2025**

Question No (9)

Question No (9)

The equation of a curve is such that $\frac{d^2y}{dx^2} = -\frac{24}{x^3}$. It is given that the curve has a stationary point at $(-2,19)$.

(a) Find an expression for $\frac{dy}{dx}$.

(b) Find the x-coordinate of the other stationary point of the curve, and determine the nature of this stationary point.

(c) Find the equation of the curve.

(d) Find the equation of the normal to the curve at the point where $\frac{dy}{dx} = -\frac{9}{4}$ and x is positive. Express your answer in the form $px + qy + r = 0$, where p, q and r are integers.

Solution:

(a)

On Next page

given equation of curve

$$\frac{d^2y}{dx^2} = -\frac{24}{x^3} \rightarrow ①$$

stationary point $(-2, 19)$

Now

$$\int \frac{d^2y}{dx^2} = \int -\frac{24}{x^3}$$

$$= \int -24\, x^{-3}$$

$$\frac{dy}{dx} = -24 \frac{x^{-3+1}}{-3+1} + c$$

$$\frac{dy}{dx} = -24 \frac{x^{-2}}{-2} + c$$

$$\frac{dy}{dx} = \frac{12}{x^2} + c \rightarrow ②$$

At stationary point $(-2, 19)$

Equation ② becomes

$$0 = \frac{12}{(-2)^2} + c \Rightarrow 0 = \frac{12}{4} + c \Rightarrow c = -3$$

Equation ② becomes

$$\frac{dy}{dx} = \frac{12}{x^2} - 3$$

(b) Now we shall find x-coordinate of second stationary point.

put

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{12}{x^2} - 3 = 0$$

$$\frac{12}{x^2} = 3$$

$$x^2 = 4$$

$$x = \pm 2$$

second stationary point is 2.

Now we check it nature

As $\frac{d^2y}{dx^2} = -\frac{24}{x^3}$

at $x = 2$

$$\frac{d^2y}{dx^2} = -\frac{24}{(2)^3}$$

$$= -\frac{24}{8} = -3$$

$$\frac{d^2y}{dx^2} < 0$$

So this is maximum stationary point.

(c) Now we shall find out the equation of the curve.

As

$$\frac{dy}{dx} = 12x^{-2} - 3$$

separate variables.

$$dy = (12x^{-2} - 3)\,dx$$

$$\int dy = \int (12x^{-2} - 3)\,dx$$

$$y = 12\,\frac{x^{-2+1}}{-2+1} - 3x + d$$

$$y = 12\,\frac{x^{-1}}{-1} - 3x + d$$

$$y = -\frac{12}{x} - 3x + d \rightarrow (1)$$

As $(-2, 19)$ lie on curve, so

$$19 = -\frac{12}{-2} - 3(-2) + d$$

$$19 = 6 + 6 + d$$

$$d = 19 - 12 = 7$$

so equation (1) becomes

$$y = -\frac{12}{x} - 3x + 7$$

(d) Given $\frac{dy}{dx} = -\frac{9}{4}$

$\Rightarrow \frac{12}{x^2} - 3 = -\frac{9}{4}$ $\quad \because \frac{dy}{dx} = \frac{12}{x^2} - 3$

$\frac{12}{x^2} = -\frac{9}{4} + 3$

$\frac{12}{x^2} = 3/4$

$x^2 = \frac{12 \times 4}{3}$

$x^2 = 16$

$x = \pm 4$

$x = 4$ (question statement)

as in part (c)

$y = \frac{-12}{x} - 3x + 7$

at $x = 4$

$y = -\frac{12}{4} - 3(4) + 7$

$= -3 - 12 + 7$

$y = -8$

point is $(4, -8)$

gradient of curve

$\frac{dy}{dx} = -9/4$ (given)

gradient of normal

$$\frac{dy}{dx} = -\frac{1}{-9/4} = \frac{4}{9} \quad \because m_1 \times m_2 = -1$$

Equation of normal by using $\frac{dy}{dx} = 4/9$

and $P(4, -8)$

$$y - y_1 = \frac{4}{9}(x - x_1)$$

$$y + 8 = \frac{4}{9}(x - 4)$$

$$9(y + 8) = 4(x - 4)$$

$$9y + 72 = 4x - 16$$

$$4x - 9y - 16 - 72 = 0$$

$$4x - 9y - 88 = 0$$