Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2025**

Question No (10)

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(a) Find the quotient and remainder when x^2 is divided by $1 + 4x^2$.

(b) Find the exact value of $\int_0^{0.5} x \tan^{-1}(2x)\, dx$

Solution:

(a)

$$4x^2+1 \overline{)\sqrt{x^2}} \quad (1/4$$

$$\underline{x^2 + 1/4}$$

$$-\frac{1}{4}$$

quotient $= 1/4$

remainder $= -\frac{1}{4}$

(b)

$$\int_0^{0.5} x \tan^{-1}(2x)\,dx \longrightarrow (1)$$

integration by parts

$$\int \underset{I}{u}\,\underset{II}{v} = u\int v - \int \frac{du}{dx}\int v$$

Trigonometry formula

$$y = \tan^{-1}(2x)$$

$$\frac{dy}{dx} = \frac{1}{1+(2x)^2}\,\frac{d}{dx}(2x)$$

$$= \frac{1}{1+4x^2}(2) = \frac{2}{1+4x^2}$$

DATE :-

Now we shall use the integration by parts

$$\int \underset{II}{x} \; \underset{I}{\tan^{-1}(2x)}\,dx = \frac{x^2}{2}\tan^{-1}(2x) - \int \frac{x^2}{2} \times \frac{1}{1+4x^2} \frac{d}{dx}(2x)$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \int \frac{x^2}{2} \times \frac{1}{1+4x^2} \quad (2)$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \int \frac{x^2}{1+4x^2}$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \int \left[\frac{1}{4} + \frac{-4}{1+4x^2}\right] dx$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \int \frac{dx}{4} + \frac{1}{4}\int \frac{dx}{1+4x^2}$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \frac{x}{4} + \frac{1}{4} \times \frac{1}{4}\int \frac{dx}{1/4 + x^2}$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \frac{x}{4} + \frac{1}{16}\int \frac{dx}{(1/2)^2 + (x)^2}$$

$$= \frac{x^2}{2}\tan^{-1}(2x) - \frac{x}{4} + \frac{1}{16} \times \frac{1}{1/2}\tan^{-1}\left(\frac{x}{1/2}\right)$$

$$= \left|\frac{x^2}{2}\tan^{-1}(2x) - \frac{x}{4} + \frac{2}{16}\tan^{-1}(2x)\right|_0^{0.5}$$

$$= \frac{(0.5)^2}{2}\tan^{-1}(2\times 0.5) - \frac{0.5}{4} + \frac{1}{8}\tan^{-1}(2\times 0) - (0)$$

$$= \frac{1}{8}\tan(1) - \frac{1}{8} + \frac{1}{8}\tan(1)$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\right) - \frac{1}{8} + \frac{1}{8}\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{32} - \frac{1}{8} + \frac{\pi}{32}$$

$$= \frac{\pi}{16} - \frac{1}{8}$$