Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2025**

Question No (11)

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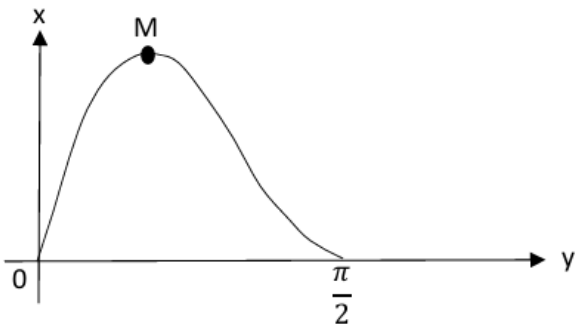
Question No (11)



The diagram shows the graph of $y = 5 \sin 2x \cos^2 x \text{ for } 0 \leq x \leq \frac{\pi}{2}$ and its maximum point M.

(a) Find the exact x-coordinate of M.

(b) By using the substitution $u = \cos x$, *find the area of the region bounded by the curve, the* $x-$ *axix between* $x = 0$ *and* $x = \frac{\pi}{4}$ *and the line* $x = \frac{\pi}{4}$.

Solution:

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(a) $y = 5 \sin 2x \cos^2 x \rightarrow (1) \quad 0 \le x \le \pi/2$

Derivative formula

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Differentiate (1) w.r.t x

$$\frac{dy}{dx} = 5\left[\sin 2x\left(2\cos x(-\sin x)\right) + \cos^2 x(\cos 2x)2\right]$$

$$= 5\left[-2\sin x \cos x \sin 2x + 2\cos 2x \cos^2 x\right]$$

$$-10 \sin x \cos x \sin 2x + 10 \cos 2x \cos^2 x$$

For maximum point $\frac{dy}{dx} = 0$

$$-10 \sin x \cos 2x (2\sin x \cos x) + 10(2\cos^2 x - 1)\cos^2 x = 0$$

$$-20 \sin^2 x \cos^2 x + 20 \cos^4 x - 10 \cos^2 x = 0$$

$$-20(1 - \cos^2 x)\cos^2 x + 20 \cos^4 x - 10 \cos^2 x = 0$$

$$-20 \cos^2 x + 20 \cos^4 x + 20 \cos^4 x - 10 \cos^2 x = 0$$

$$40 \cos^4 x - 30 \cos^2 x = 0$$

$$10 \cos^2 x (4 \cos^2 x - 3) = 0$$

$$\Rightarrow 4 \cos^2 x - 3 = 0$$

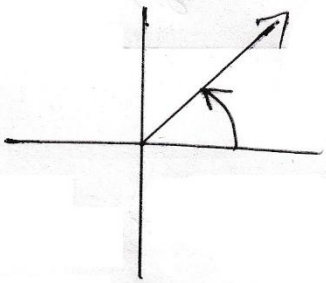
$$4 \cos^2 x = 3$$

$$\cos^2 x = 3/4$$

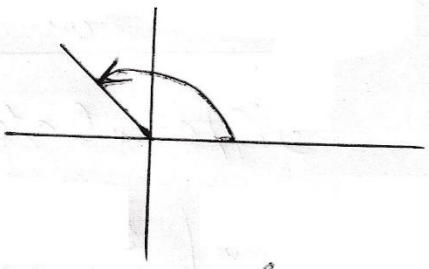
$$\cos x = \pm\sqrt{3/4}$$

$$\cos x = \pm \sqrt{3}/2$$

$\cos x = \sqrt{3}/2$



$\cos x = -\sqrt{3}/2$



ignore as interval is $0 \leq x \leq \pi/2$

$x = \cos^{-1}(\sqrt{3}/2)$

$x = \pi/6$

Exact coordinate of x is $x = \pi/6$

(b)

$$y = 5 \sin 2x \cos^2 x \qquad 0 \leq x \leq \pi/2$$

$$= 5(2\sin x \cos x)\cos^2 x$$

$$y = 10 \sin x \cos^3 x$$

By given substitution

$$u = \cos x$$

$$du = -\sin x \, dx$$

$$-du = \sin x \, dx$$

DATE :-

Given limit is

$x=0$ and $x=\pi/4$

Now These limit change w.r.t u variable,

As

$u=\cos x$

$u=\cos 0 \quad \because x=0$

$u=1$

$u=\cos \pi/4 \quad \because x=\pi/4$

$u=\frac{1}{\sqrt{2}}$

Now area of region bounded by the curve

$$A=\int_{1}^{1/\sqrt{2}} y\,dx$$

$$=-10\int_{1}^{1/\sqrt{2}} u^3\,du$$

$$=-10\left|\frac{u^4}{4}\right|_{1}^{1/\sqrt{2}}$$

$$= -\frac{10}{4}\left[\left(\frac{1}{\sqrt{2}}\right)^4 - (1)^4\right]$$

$$= -\frac{10}{4}\left[\frac{1}{4} - 1\right]$$

$$= -\frac{10}{4}\left[-\frac{3}{4}\right] = \frac{30}{16}$$

$$= \frac{15}{8}$$

$$= \frac{15}{8}$$