

Cambridge International AS & A Level

Mathematics

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Paper 3 Pure Mathematics 3

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Question No (5)

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Question No (5)

The square roots of $-1 - 4\sqrt{5}i$ can be expressed in the Cartesian form $x + iy$, where x and y are real and exact.

By first forming a quartic equation in x or y , find the square roots of $-1 - 4\sqrt{5}i$ in exact Cartesian form.

Solution:

According to given condition

$$\sqrt{-1 - 4\sqrt{5}i} = x + iy$$

Squaring both sides

$$-1 - 4\sqrt{5}i = (x + iy)^2$$

$$= x^2 + 2x(iy) + (iy)^2$$

$$= x^2 + 2ixy + i^2 y^2$$

$$= x^2 + 2xyi - y^2 \quad \because i^2 = -1$$

$$-1 - 4\sqrt{5}i = x^2 - y^2 + i2xy$$

comparing real and imaginary parts

$$x^2 - y^2 = -1 \rightarrow (1)$$

$$2xy = -4\sqrt{5} \rightarrow (2)$$

$$\Rightarrow y = \frac{-4\sqrt{5}}{2x} = -\frac{2\sqrt{5}}{x} \rightarrow (3)$$

$$\text{put } y = -\frac{2\sqrt{5}}{x} \text{ in (1)}$$

$$x^2 - \left(-\frac{2\sqrt{5}}{x}\right)^2 = -1$$

$$x^2 - \frac{4(5)}{x^2} = -1$$

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$$x^4 - 20 = -x^2$$

$$x^4 + x^2 - 20 = 0 \rightarrow (4)$$

$$\text{put } x^2 = t \text{ in (4)}$$

$$\Rightarrow t^2 + t - 20 = 0$$

factorize

$$t^2 + 5t - 4t - 20 = 0$$

$$t(t+5) - 4(t+5) = 0$$

$$(t+5)(t-4) = 0$$

$$t = -5, \quad t = 4$$

$$\Rightarrow x^2 = -5$$

$$x^2 = 4$$

$$x = \pm \sqrt{5} \text{ (undefined)}$$

$$x = \pm 2$$

$$\text{put } x = \pm 2 \text{ in (3)}$$

$$y = \frac{-2\sqrt{5}}{\pm 2}$$

$$y = \pm (-\sqrt{5})$$

so the square root is

$$\sqrt{-1-4\sqrt{5}i} = x + iy$$

$$= \pm 2 - (\pm \sqrt{5}i)$$

$$= \pm (2 - \sqrt{5}i)$$

