

Cambridge International AS & A Level

Mathematics

9709/32

Paper 3 Pure Mathematics 3

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Question No (6)

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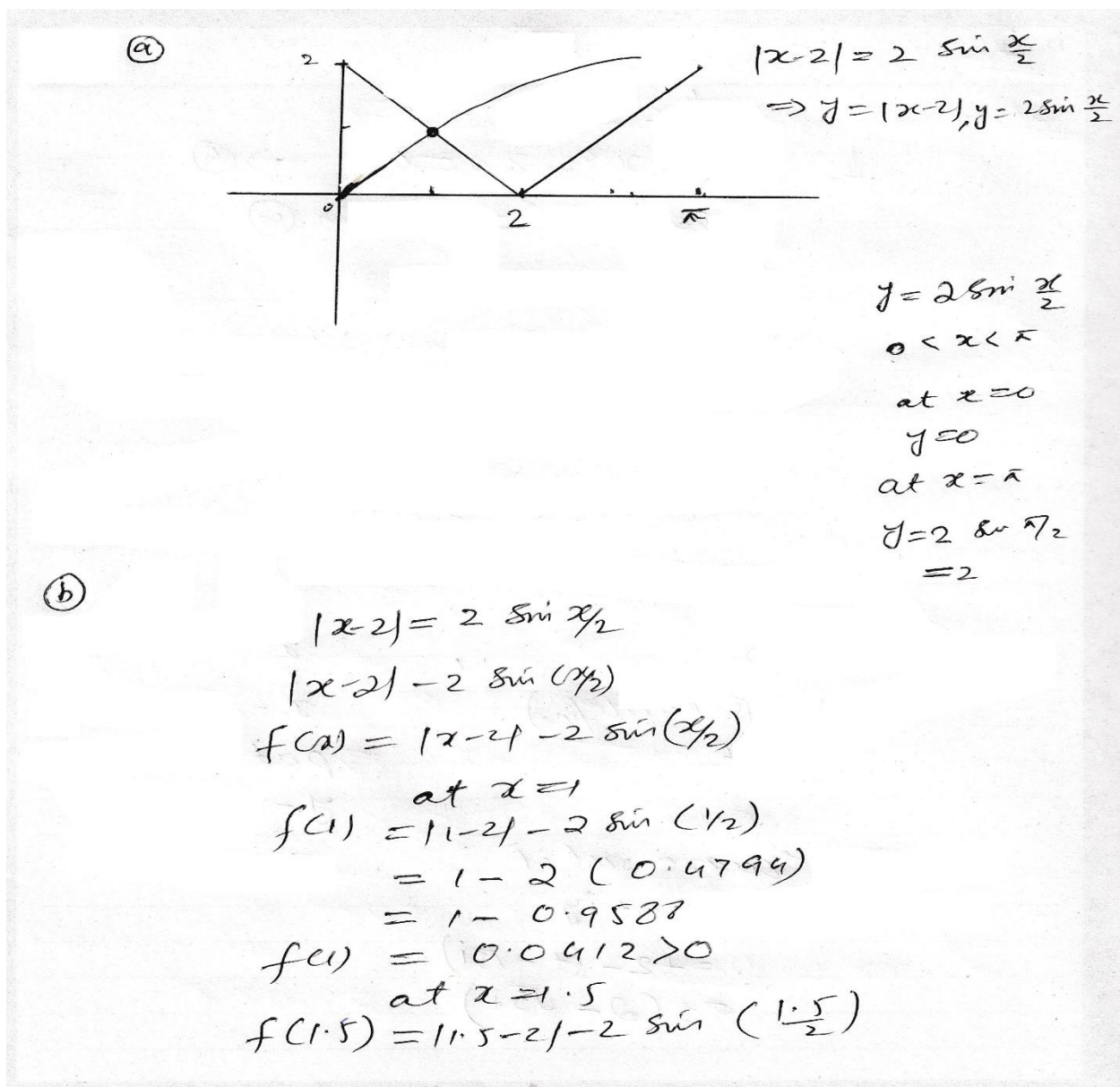
(a) By sketching a suitable pair of graphs, show that the equation

$$|x - 2| = 2 \sin \frac{x}{2}$$

has only one root in the interval $0 < x < \pi$.

(b) Show by calculation that this root lies between 1 and 1.5.

(c) Use the iterative formula $x_{n+1} = 2 - 2 \sin \frac{1}{2}x_n$ with an initial value of 1.03 to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

Solution:

$$= 0.5 - 2(\sin(0.75))$$

$$= 0.5 - 1.3632$$

$$f(1.5) = -0.8632 < 0$$

As sign of the function change between $f(1)$ and $f(1.5)$, so root lie between 1 and 1.5.

©

Given

$$x_{n+1} = 2 - 2 \sin \frac{1}{2} x_n, \quad x_0 = 1.03$$

at $n=0$

$$x_1 = 2 - 2 \sin \frac{1}{2} x_0$$

$$= 2 - 2 \sin \frac{1}{2} (1.03) \quad \because x_0 = 1.03$$

$$= 2 - 2 \sin(0.515)$$

$$x_1 = 1.0149$$

at $n=1$

$$x_2 = 2 - 2 \sin \frac{x_1}{2}$$

$$= 2 - 2 \sin \left(\frac{1.0149}{2} \right) \quad \because x_1 = 1.0149$$

$$= 2 - 2 \sin(0.5074)$$

$$x_2 = 1.0281$$

at $n=2$

$$\begin{aligned}
 x_3 &= 2 - 2 \sin\left(\frac{x_2}{2}\right) \\
 &= 2 - 2 \sin\left(\frac{1.0281}{2}\right) \\
 &= 2 - 2 \sin(0.5140) \\
 x_3 &= 1.0166
 \end{aligned}$$

at $n=3$

$$\begin{aligned}
 x_4 &= 2 - 2 \sin\left(\frac{x_3}{2}\right) \\
 &= 2 - 2 \sin\left(\frac{1.0166}{2}\right) = x_3 = 1.0166 \\
 &= 2 - 2 \sin(0.5083) \\
 x_4 &= 1.0266
 \end{aligned}$$

at $n=4$

$$\begin{aligned}
 x_5 &= 2 - 2 \sin\left(\frac{x_4}{2}\right) \\
 &= 2 - 2 \sin\left(\frac{1.0266}{2}\right) \\
 &= 2 - 2 \sin(0.5133) \\
 x_5 &= 1.0281
 \end{aligned}$$

So the root is 1.02

