Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2025**

Question No (7)

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(a) Express $7\ \sin\theta + 24\ \cos\theta$ *in the form* $R\cos(\theta-\alpha)$ *, where* $R > 0$ *and* $0 < \alpha < \frac{\pi}{2}$ .Give the value of α correct to 4 decimal places.

(b) Hence solve the equation $7\ \sin\frac{x}{3} + 24\ \cos\frac{x}{3} = 24.5$ *for* $0 < x < \pi$.

Solution:

DATE :-

ⓐ let $7\sin\theta + 24\cos\theta \equiv R\cos(\theta-\alpha)$

Trigonometry Formula

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$7\sin\theta + 24\cos\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$7\sin\theta + 24\cos\theta = R\cos\theta . \cos\alpha + R\sin\theta\sin\alpha$

comparing on both sides.

$R\sin\alpha = 7 \rightarrow (i)$

$R\cos\alpha = 24 \rightarrow (ii)$

From (i) & (ii)

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{7}{24}$

$\tan\alpha = \frac{7}{24}$

$\alpha = \tan^{-1}\left(\frac{7}{24}\right)$

$= \tan^{-1}(0.2916)$

$\alpha = 0.2837$

From (1) & (2)

$R^2\sin^2\alpha + R^2\cos^2\alpha = (7)^2 + (24)^2$

$R^2(\sin^2\alpha + \cos^2\alpha) = 49 + 576$

$R^2 = 625 \Rightarrow R = 25$

$\therefore \quad 7\sin\theta + 24\cos\theta = R\cos(\theta - \alpha)$

$= 25\cos(\theta - 0.2837)$

(b)

$7\sin\frac{1}{3}x + 24\cos\frac{1}{3}x = 24.5$ for $0 < x < \pi$

put $\frac{x}{3} = \theta$

$\Rightarrow 7\sin\theta + 24\cos\theta = 24.5$

$\Rightarrow 25\cos(\theta - 0.2837) = 24.5$ from part (a)

$\Rightarrow 25\cos(\frac{x}{3} - 0.2837) = 24.5$ as $\theta = x/3$

$\cos(\frac{x}{3} - 0.2837) = \frac{24.5}{25}$

$\frac{x}{3} - 0.2837 = \cos^{-1}\left(\frac{24.5}{25}\right)$

$= \cos^{-1}(0.98)$

$= 0.2003$

$\frac{x}{3} = 0.2003 + 0.2837$

$x = 3\,(0.2003 + 0.2837)$

$x = 1.45^{\circ}$