Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2025**

Question No (8)

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The variables x and θ satisfy the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (4x+3)\cos 2\theta,$$

$$and\ x = 0\ when\ \theta = \frac{\pi}{12}$$

. Solve the differential equation and obtain an expression for x in terms of θ.

Solution:

DATE :-

$$\sin 2\theta \frac{dx}{d\theta} = (4x+3)\cos 2\theta$$

$$\frac{dx}{4x+3} = \frac{\cos 2\theta}{\sin 2\theta}\, d\theta$$

$$\frac{1}{4}\int \frac{4\, dx}{4x+3} = \frac{1}{2}\int \frac{\cos 2\theta \; 2d\theta}{\sin 2\theta}$$

$$\frac{1}{4}\ln(4x+3) = \frac{1}{2}\ln \sin 2\theta + c \rightarrow (1)$$

Given at $x=0$ when $\theta = \frac{\pi}{12}$

$$\frac{1}{4}\ln(4(0)+3) = \frac{1}{2}\ln \sin 2\times\frac{\pi}{12} + c$$

$$\frac{1}{4}\ln 3 = \frac{1}{2}\ln \sin\frac{\pi}{6} + c$$

$$\frac{1}{2^2}\ln 3 = \frac{1}{2}\ln\left(\frac{1}{2}\right) + c$$

$$2^{-2}\ln 3 = 2^{-1}\ln(2^{-1}) + c$$

$$\ln(3)^{2^{-2}} = \ln(2^{-1})^{2^{-1}} + c$$

$$\ln(3)^{2^{-2}} - \ln(2^{-1})^{2^{-1}} = c$$

$$\ln\left[\frac{(3)^{2^{-2}}}{(2^{-1})^{2^{-1}}}\right] = c$$

$$\ln\left[\frac{3^{1/4}}{\left(\frac{1}{2}\right)^{1/2}}\right] = c$$

$$\ln\left[\frac{1.3160}{0.7071}\right] = c$$

$$c = \ln(1.8611)$$

put c in equation (1)

$$\frac{1}{4}\ln(4x+3) = \frac{1}{2}\ln(\sin 2\theta) + \ln(1.8611)$$

$$\ln(4x+3) = 2\ln\sin 2\theta + 4\ln(1.8611)$$

$$\ln(4x+3) = \ln(\sin^2 2\theta) + \ln(1.8611)^4$$

$$\ln(4x+3) = \ln\left[(\sin^2 2\theta)(1.8611)^4\right]$$

$$4x+3 = 12\sin^2 2\theta$$

$$4x = 12\sin^2 2\theta - 3$$

$$x = \frac{12\sin^2 2\theta - 3}{4}$$