Cambridge International AS & A Level

Mathematics 9709/32

Paper 3 Pure Mathematics 3 **May/June 2025**

Question No (9)

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Question No (9)

With respect to the origin O, the points A, B and C have position vectors given by

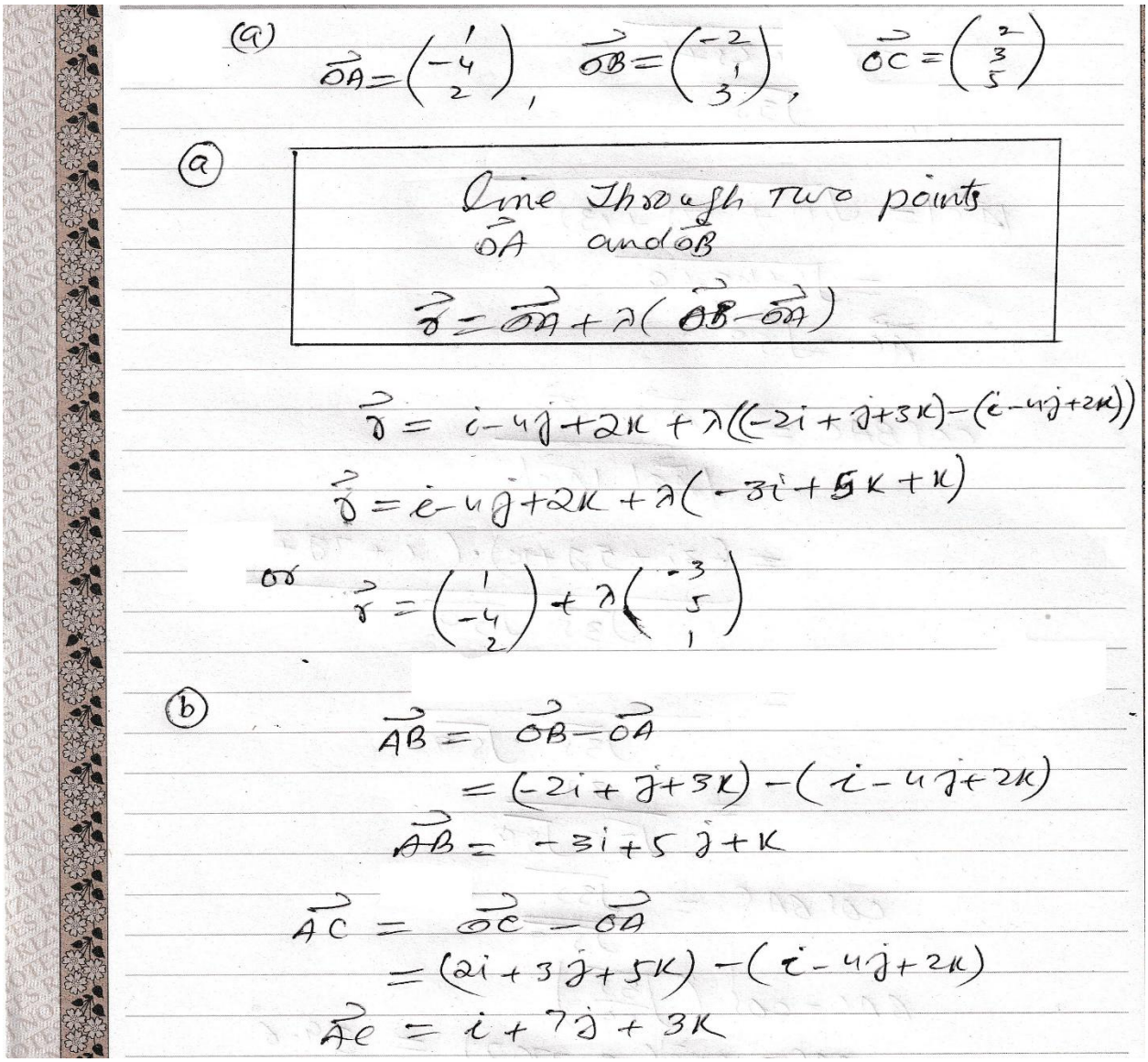
$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \overrightarrow{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

(a) Find a vector equation for the line through A and B.

(b) Using a scalar product, find the exact value of cos BAC.

(c) Hence find the exact area of triangle ABC.

Solution:



$$|\vec{AB}| = \sqrt{(-3)^2 + (5)^2 + (1)^2}$$

$$= \sqrt{9 + 25 + 1}$$

$$|\vec{AB}| = \sqrt{35}$$

$$|\vec{AC}| = \sqrt{(1)^2 + (7)^2 + (3)^2}$$

$$= \sqrt{1 + 49 + 9}$$

$$\vec{AC} = \sqrt{59}$$

$$\cos BAC = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}|\,|\vec{AC}|}$$

$$= \frac{(-3i + 5j + k) \cdot (i + 7j + 3k)}{\sqrt{35}\ \sqrt{59}}$$

$$\cos BAC = \frac{-3 + 35 + 3}{\sqrt{35}\ \sqrt{59}}$$

$$= \frac{35}{\sqrt{35}\ \sqrt{59}}$$

$$\cos BAC = \frac{\sqrt{35}}{\sqrt{59}} \checkmark$$

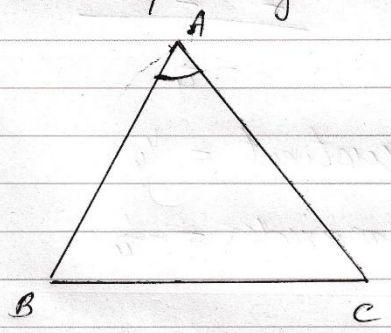
$$BAC = \cos^{-1}\left(\sqrt{\frac{35}{59}}\right)$$

$$BAC = \cos^{-1}(0.7702) = 39.6^\circ$$

DATE :-

(c)

Area of triangle ABC



$\Delta = \frac{1}{2} |\vec{AB}| |\vec{AC}| \sin B\hat{A}C$

$\Delta = \frac{1}{2} \sqrt{35} \cdot \sqrt{59} \sin B\hat{A}C$

$= \frac{1}{2} \sqrt{35} \cdot \sqrt{59} \sqrt{1 - \cos^2 B\hat{A}C}$

$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - (\cos B\hat{A}C)^2}$

$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - \left(\frac{\sqrt{35}}{59}\right)^2}$

$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - \frac{35}{59}}$

$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{\frac{59 - 35}{59}} = \frac{1}{2} \sqrt{35} \sqrt{59} \frac{\sqrt{24}}{\sqrt{59}}$

$\Delta = \frac{1}{2} \sqrt{35} \times \sqrt{24}$

$\Delta = \frac{\sqrt{840}}{2} = \frac{\sqrt{2 \times 2 \times 210}}{2} = \frac{2\sqrt{210}}{2} = \sqrt{210}$