

Cambridge International AS & A Level

Mathematics

9709/32

Paper 3 Pure Mathematics 3

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Question No (9)

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Question No (9)

With respect to the origin O, the points A, B and C have position vectors given by

$$\vec{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \text{ and } \vec{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

- (a) Find a vector equation for the line through A and B.
 (b) Using a scalar product, find the exact value of $\cos BAC$.
 (c) Hence find the exact area of triangle ABC.

Solution:

(a) $\vec{OA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$

(a)

Line Through two points
 $\vec{OA}$ and $\vec{OB}$

$$\vec{r} = \vec{OA} + \lambda(\vec{OB} - \vec{OA})$$

$$\vec{r} = i - 4j + 2k + \lambda((-2i + j + 3k) - (i - 4j + 2k))$$

$$\vec{r} = i - 4j + 2k + \lambda(-3i + 5j + k)$$

or
$$\vec{r} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$$

(b)

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (-2i + j + 3k) - (i - 4j + 2k)$$

$$\vec{AB} = -3i + 5j + k$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$= (2i + 3j + 5k) - (i - 4j + 2k)$$

$$\vec{AC} = i + 7j + 3k$$

$$|\vec{AB}| = \sqrt{(-3)^2 + (5)^2 + (1)^2}$$

$$= \sqrt{9 + 25 + 1}$$

$$|\vec{AB}| = \sqrt{35}$$

$$|\vec{AC}| = \sqrt{(1)^2 + (7)^2 + (3)^2}$$

$$= \sqrt{1 + 49 + 9}$$

$$|\vec{AC}| = \sqrt{59}$$

$$\cos BAC = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$= \frac{(-3i + 5j + k) \cdot (i + 7j + 3k)}{\sqrt{35} \sqrt{59}}$$

$$\cos BAC = \frac{-3 + 35 + 3}{\sqrt{35} \sqrt{59}}$$

$$= \frac{35}{\sqrt{35} \sqrt{59}}$$

$$\cos BAC = \frac{\sqrt{35}}{\sqrt{59}} \quad \checkmark$$

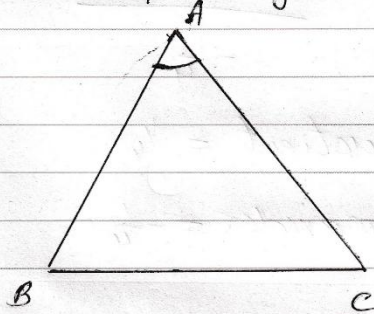
$$BAC = \cos^{-1} \left(\frac{\sqrt{35}}{\sqrt{59}} \right)$$

$$BAC = \cos^{-1}(0.7702) = 39.6^\circ$$

DATE :-

(C)

Area of triangle ABC



$$\Delta = \frac{1}{2} |\vec{AB}| |\vec{AC}| \sin \hat{BAC}$$

$$\Delta = \frac{1}{2} \sqrt{35} \cdot \sqrt{59} \sin \hat{BAC}$$

$$= \frac{1}{2} \sqrt{35} \cdot \sqrt{59} \sqrt{1 - \cos^2 \hat{BAC}}$$

$$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - (\cos \hat{BAC})^2}$$

$$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - \left(\sqrt{\frac{35}{59}}\right)^2}$$

$$= \frac{1}{2} \sqrt{35} \sqrt{59} \sqrt{1 - \frac{35}{59}}$$

$$= \frac{1}{2} \sqrt{35} \sqrt{59} \frac{\sqrt{59 - 35}}{59} = \frac{1}{2} \sqrt{35} \sqrt{59} \frac{\sqrt{24}}{\sqrt{59}}$$

$$\Delta = \frac{1}{2} \sqrt{35} \times \sqrt{24}$$

$$\Delta = \frac{\sqrt{840}}{2} = \frac{\sqrt{2 \times 2 \times 210}}{2} = \frac{2\sqrt{210}}{2} = \sqrt{210}$$

