Cambridge International AS & A Level

Mathematics 9709/52

Paper 5 Probability & Statistics 1 **May/June 2025**

Question No (7)

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Kestrels are birds whose adult wingspans are normally distributed with mean 74.8 cm and standard deviation 3.2 cm. A random sample of 120 adult kestrels is selected.

(a) How many of these 120 adult kestrels would you expect to have wingspan between 72.4 cm and 76.3 cm?

The masses of adult kestrels are normally distributed with mean μ kg and standard deviation σ kg. It is known that 20% of adult kestrels have mass greater than 0.202 kg and 28% have mass less than 0.185 kg.

(b) Find the value of μ and the value of σ.

10 adult kestrels are selected at random.

(c) Find the probability that fewer than 3 have masses greater than 0.202 kg.

Solution:

(a) Given data

$\mu = 74.8, \quad \sigma = 3.2, \quad n = 120$

Let x be the variable of normal distribution.

By given condition

$P(72.4 < X < 76.3)$

By standardize to z-score

$$P\left(\frac{72.4 - 74.8}{3.2} < \frac{X-\mu}{\sigma} < \frac{76.3 - 74.8}{3.2}\right)$$

$$P(-0.75 < z < 0.4687) \quad \because z = \frac{X-\mu}{\sigma}$$

$$= \Phi(0.4687) - \Phi(-0.75)$$

$$= \Phi(0.4687) - [1 - \Phi(0.75)]$$

$$= \Phi(0.4687) - 1 + \Phi(0.75)$$

$$= \Phi(0.75) + \Phi(0.4687) - 1$$

$$= 0.7734 + 0.6804 - 1$$

$$= 0.4538$$

Expected number $= 0.4538 \times 120$

$= 54$

(b) By the given condition

$$P(X > 0.202) = 20\%$$

$$P(X > 0.202) = \frac{20}{100} = 0.2$$

By standardize z-score

$$P\left(\frac{X-\mu}{\sigma} > \frac{0.202-\mu}{\sigma}\right) = 0.2$$

$$P\left(Z > \frac{0.202-\mu}{\sigma}\right) = 0.2$$

$$P\left(Z < \frac{0.202-\mu}{\sigma}\right) = 1-0.2$$

$$P\left(Z < \frac{0.202-\mu}{\sigma}\right) = 0.8$$

$$\phi\left(\frac{0.202-\mu}{\sigma}\right) = 0.8$$

$$\frac{0.202-\mu}{\sigma} = \phi^{-1}(0.8)$$

$$\frac{0.202-\mu}{\sigma} = 0.8416$$

$$0.202-\mu = \sigma(0.8416) \rightarrow (1)$$

DATE :-

By second condition of Question

$P(X < 0.185) = 28\%$

$P(X < 0.185) = 0.28$

By standardize z-score

$P\left(\frac{X-\mu}{\sigma} < \frac{0.185-\mu}{\sigma}\right) = 0.28$

$P\left(Z < \frac{0.185-\mu}{\sigma}\right) = 0.28$

$\Phi\left(-\left(\frac{0.185-\mu}{\sigma}\right)\right) = 1-0.28$

$\Phi\left(-\left(\frac{0.185-\mu}{\sigma}\right)\right) = 0.72$

$-\left(\frac{0.185-\mu}{\sigma}\right) = \Phi^{-1}(0.72)$

$= 0.583$

$\frac{0.185-\mu}{\sigma} = -0.583$

$0.185-\mu = -0.583\sigma \rightarrow ②$

solving Equation ① & ②

$$0.185 - \mu = -0.583\,\sigma$$
$$\underline{-0.202 \mp \mu = -0.8416\,\sigma}$$
$$-0.017 = -1.4246\,\sigma$$
$$\sigma = 0.0119$$

put $\sigma = 0.0119$ in Equation (2)

$$0.185 - \mu = -0.583\,(0.0119)$$
$$0.185 - \mu = -0.006937$$
$$\mu = 0.185 + 0.00693$$
$$\mu = 0.1919$$
$$\mu = 0.192$$

(c)

we already know that

$$P(X > 0.202) = 0.20$$

As $p = 0.20 \Rightarrow q = 1 - p = 0.80$

and $n = 10$

By given condition

$$P(X < 3) = P(0, 1, 2)$$

$P(X<3) = {}^{10}C_0\, p^0 q^{10} + {}^{10}C_1\, p^1 q^9 + {}^{10}C_2\, p^2 q^8$

$$\boxed{\begin{array}{l} {}^{10}C_x\, (p)^x\, (1-p)^{10-x} \\ \text{or} \\ {}^{10}C_x\, (p)^x\, q^{10-q} \end{array}}$$

$P(X<3) = (0.8)^{10} + 10\,(0.2)^1 (0.8)^9 + \frac{10\times 9}{2\times 1}(0.2)^2 (0.8)^8$

$= 0.1073 + 0.2684 + 0.3019$

$= 0.6776$

$P(X<3) = 0.678$