Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (10)

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The line $y = \frac{x}{k} + k$,where k is a constant, is a tangent to the curve $4y = x^2$ at the point P. Find

(i) the value of k,

(ii) the coordinates of P.

Solution

Equation of line

$$y = \frac{x}{k} + k \longrightarrow (1)$$

Equation of curve

$$4y = x^2 \longrightarrow (2)$$

(ii)

solving (1) & (2)

$$4\left(\frac{x}{k} + k\right) = x^2$$

$$4\frac{x}{k} + 4k = x^2$$

$$4x + 4k^2 = kx^2$$

$$kx^2 - 4x - 4k^2 = 0$$

As line is tangent to the curve

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4(k)(-4k^2) = 0$$

$$16 + 16k^3 = 0$$

$$16k^3 = -16$$

$$k^3 = -1$$

$$(k)^3 = (-1)^3$$

$$\therefore k = -1$$

(ii) Equation of line

$$y = \frac{x}{k} + k$$

$$y = \frac{x}{-1} - 1 \qquad \because k = -1$$

$$y = -x - 1 \longrightarrow (3)$$

put $y = -x - 1$ in (2)

$$4(-x-1) = x^2$$

$$-4x - 4 = x^2$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)^2 = 0$$

$$\Rightarrow x + 2 = 0$$

$$x = -2$$

put $x = -2$ in (3)

$$y = -(-2) - 1$$

$$= 2 - 1$$

$$y = 1$$

$\therefore$ point P is $(-2, 1)$