

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (12)

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Question No (12)

A curve has equation $y = 2x^2 - 3x$

- (i) Find the set of values of x for which $y > 9$.
- (ii) Express $2x^2 - 3x$ in the form $a(x + b)^2 + c$, where a, b and c are constants, And state the coordinates of the vertex of the curve.

The functions f and g are defined for all real values of x by

$$f(x) = 2x^2 - 3x \text{ and } g(x) = 3x + k,$$

where k is a constant.

- (iii) Find the value of k for which the equation $gf(x) = 0$ has equal roots.

Solution

Equation of curve
 $y = 2x^2 - 3x \rightarrow \textcircled{1}$

condition
 $y > 9$
 $\Rightarrow 2x^2 - 3x > 9$ $\leftarrow y = 2x^2 - 3x$

$2x^2 - 3x - 9 > 0$
 $2x^2 - 6x + 3x - 9 > 0$ factorization
 $2x(x-3) + 3(x-3) > 0$
 $(x-3)(2x+3) > 0$

critical value
 $x-3=0, 2x+3=0$
 $x=3, x=-3/2$

As in $2x^2 - 3x - 9 > 0$
 $a=2 > 0$ so cup face up

$ax^2 + bx + c$
 $a > 0$
 cup face up

Due to greater inequality i.e

$2x^2 - 3x - 9 > 0$, we shall take
outer value of graph.

$$x > 3, \quad x < -\frac{3}{2}$$

(ii)

Now we express $2x^2 - 3x$ in form
 $a(x+b)^2 + c$

$$\begin{aligned} & 2x^2 - 3x \\ &= 2 \left[x^2 - \frac{3}{2}x \right] \\ &= 2 \left[x^2 - \frac{3}{2} \times \frac{3}{2}x \right] \\ &= 2 \left[x^2 - 2 \left(\frac{3}{4} \right)x \right] \\ &= 2 \left[x^2 - 2 \left(\frac{3}{4} \right)x + \left(\frac{3}{4} \right)^2 - \left(\frac{3}{4} \right)^2 \right] \\ &= 2 \left[x^2 - 2 \left(\frac{3}{4} \right)x + \left(\frac{3}{4} \right)^2 \right] - 2 \left(\frac{3}{4} \right)^2 \\ &= 2 \left(x - \frac{3}{4} \right)^2 - 2 \times \frac{9}{16} \\ &= 2 \left(x - \frac{3}{4} \right)^2 - \frac{9}{8} \\ &\text{Vertex is } \left(\frac{3}{4}, -\frac{9}{8} \right) \end{aligned}$$

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Given

$$f(x) = 2x^2 - 3x \quad \text{and} \quad g(x) = 3x + k$$

$$g(f(x)) = 3(f(x)) + k$$

$$= 3(2x^2 - 3x) + k \quad \text{4. } f(x) = 2x^2 - 3x$$

$$= 6x^2 - 9x + k$$

given condition

$g(f(x))$ has equal roots

$$\Rightarrow b^2 - 4ac = 0$$

$$(-9)^2 - 4(6)(k) = 0$$

$$81 - 24k = 0$$

$$-24k = -81$$

$$k = \frac{-81}{-24}$$

$$k = \frac{81}{24}$$