Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (12)

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A curve has equation $y = 2\,x^2 - 3x$

(i) Find the set of values of x for which y > 9.

(ii) Express $2\,x^2 - 3x$ in the form $a\,(x+b)^2 + c$,where a,b and c are constants, And state the coordinates of the vertex of the curve.

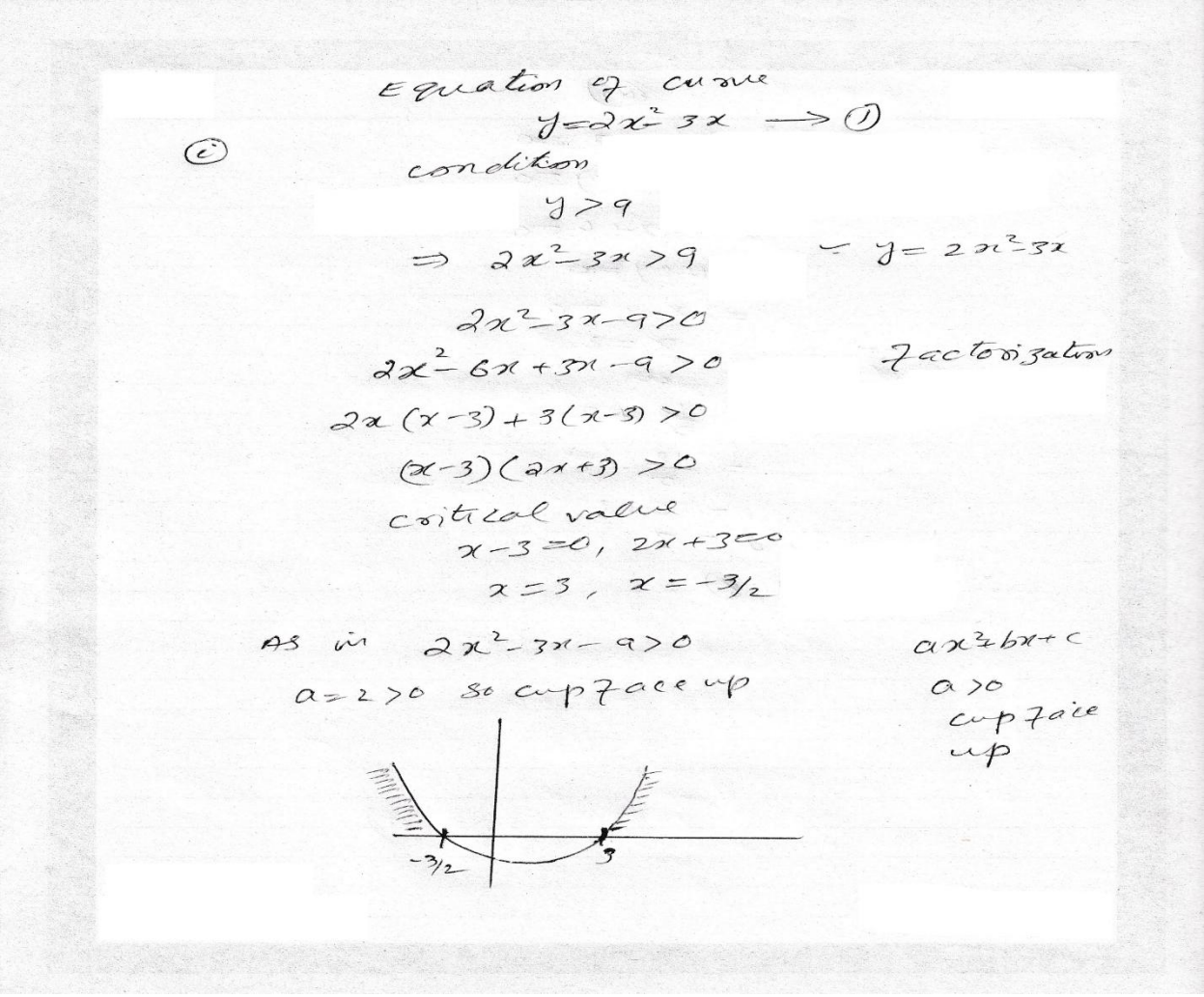
The functions f and g are defined for all real values of x by

$$f(x) = 2\,x^2 - 3x \;\; and \; g(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $gf(x) = 0$ has equal roots.

Solution



Due to greater inequality i.e
$2x^2 - 3x - 9 > 0$, we shall take outer value of graph.

$$x > 3, \quad x < -3/2$$

(ii)

Now we express $2x^2 - 3x$ in form
$a(x+b)^2 + c$

$$2x^2 - 3x$$
$$= 2\left[x^2 - \frac{3}{2}x\right]$$
$$= 2\left[x^2 - \frac{2}{2} \times \frac{3}{2}x\right]$$
$$= 2\left[x^2 - 2\left(\frac{3}{4}\right)x\right]$$
$$= 2\left[x^2 - 2(3/4)x + (3/4)^2 - (3/4)^2\right]$$
$$= 2\left[x^2 - 2(3/4)x + (3/4)^2\right] - 2(3/4)^2$$
$$= 2(x - 3/4)^2 - 2 \times \frac{9}{16}$$
$$= 2(x - 3/4)^2 - \frac{9}{8}$$

Vertex is $(3/4, -9/8)$

(iii) Given

$f(x) = 2x^2 - 3x$ and $g(x) = 3x + K$

$g(f(x)) = 3(f(x)) + K$

$= 3(2x^2 - 3x) + K$ ∵ $f(x) = 2x^2 - 3x$

$= 6x^2 - 9x + K$

given condition

$g(f(x))$ has equal roots

$\Rightarrow b^2 - 4ac = 0$

$(-9)^2 - 4(6)(K) = 0$

$81 - 24K = 0$

$-24K = -81$

$K = -\frac{81}{-24}$

$K = \frac{81}{24}$