Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (13)

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A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$.

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve.

Solution

Equation of curve

$y = 2x^2 - 6x + 5 \longrightarrow (1)$

(i) given condition

$y > 13$

$\Rightarrow 2x^2 - 6x + 5 > 13$ $\quad$ $\because y = 2x^2 - 6x + 5$

$2x^2 - 6x + 5 - 13 > 0$

$2x^2 - 6x - 8 > 0$

$2(x^2 - 3x - 4) > 0$

$\Rightarrow (x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$ $\quad$ Factorization

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

critical values

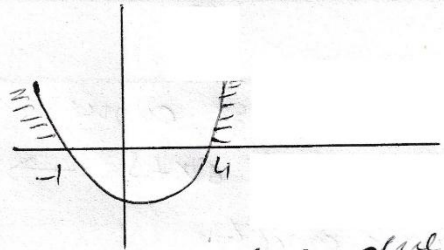
$x - 4 = 0, \quad x + 1 = 0$

$x = 4, \quad x = -1$

Due to

$x^2 - 3x - 4 > 0$ $\quad$ $ax^2 + bx + c$

$a > 0$ $\quad$ $a > 0$ it has cup face up



taking outer value due to

$$x^2 - 3x + 4 > 0$$

so The values of x are

$$x < -1, \ x > 4$$

(ii) Equation of line

$$y = 2x + K \rightarrow (1)$$

Equation of curve

$$y = 2x^2 - 6x + 5 \rightarrow (2)$$

solving (1) & (2)

$$2x + K = 2x^2 - 6x + 5$$

$$2x^2 - 6x + 5 - 2x + K = 0$$

$$2x^2 - 8x + 5 - K = 0$$

As line is tangent to curve, so

$$b^2 - 4ac = 0$$

$$(-8)^2 - 4(2)(5-K) = 0$$

$$64 - 8(5-K) = 0$$

$$64 - 40 + 8K = 0$$

$$24 + 8k = 0$$

$$8k = -24$$

$$k = -\frac{24}{8}$$

$$k = -3$$