Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (15)

Question No (15)

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is

$y + x = k$, where k is a constant.

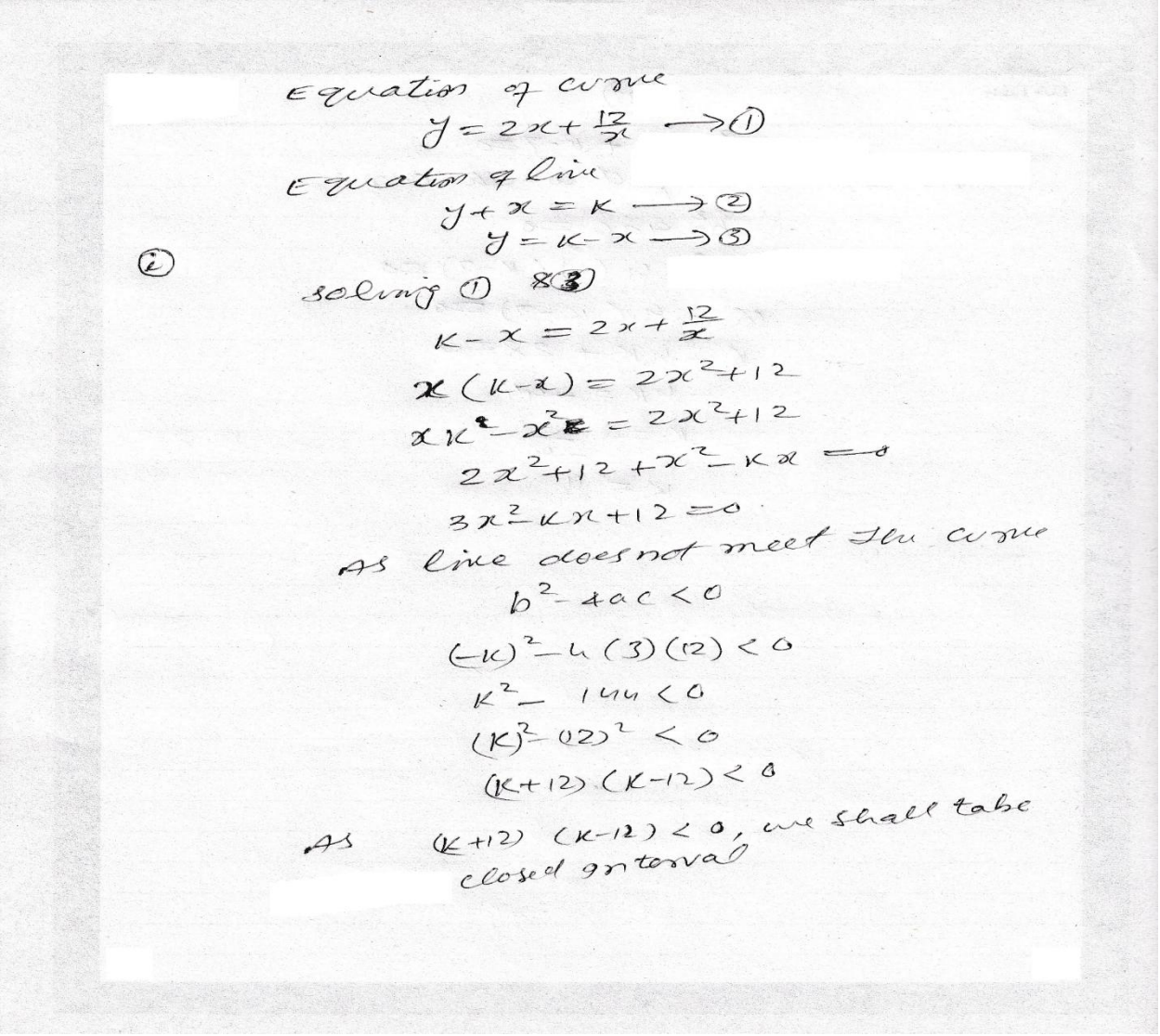
(i) Find the set of values of k for which the line does not meet the curve.

In the case where k=15, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B.

(iii) Find the equation of the perpendicular bisector of the line joining A and B

Solution



Equation of curve

$y = 2x + \frac{12}{x} \longrightarrow (1)$

Equation of line

$y + x = k \longrightarrow (2)$

$y = k - x \longrightarrow (3)$

(i)

solving (1) & (3)

$k - x = 2x + \frac{12}{x}$

$x(k-x) = 2x^2 + 12$

$xk - x^2 = 2x^2 + 12$

$2x^2 + 12 + x^2 - kx = 0$

$3x^2 - kx + 12 = 0$

As line does not meet the curve

$b^2 - 4ac < 0$

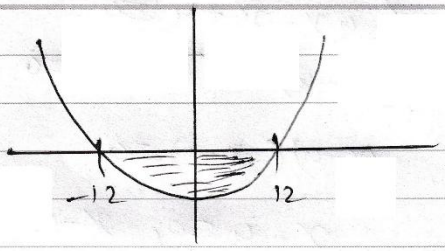
$(-k)^2 - 4(3)(12) < 0$

$k^2 - 144 < 0$

$(k)^2 - (12)^2 < 0$

$(k+12)(k-12) < 0$

As $(k+12)(k-12) < 0$, we shall take closed interval



$-12 < K < 12$

(ii)

Equation of curve

$$y = 2x + \frac{12}{x} \rightarrow (1)$$

Equation of line

$$y + x = K$$

$$y = K - x$$

As $K = 15$

$$\Rightarrow y = 15 - x \rightarrow (2)$$

Solving (1) & (2)

$$15 - x = 2x + \frac{12}{x}$$

$$x(15 - x) = 2x^2 + 12$$

$$15x - x^2 = 2x^2 + 12$$

$$2x^2 + 12 + x^2 - 15x = 0$$

$$3x^2 - 15x + 12 = 0$$

$$3(x^2 - 5x + 4) = 0$$

$$x^2 - 5x + 4 = 0$$

By factorization

$x^2 - 4x - x + 4 = 0$

$x(x-4) - 1(x-4) = 0$

$(x-4)(x-1) = 0$

$x - 4 = 0, \quad x - 1 = 0$

$x = 4, \quad x = 1$

when $x = 4$

Equation ② becomes

$y = 15 - 4$

$y = 11$

$B(4, 11)$

when $x = 1$

Equation ② becomes

$y = 15 - 1$

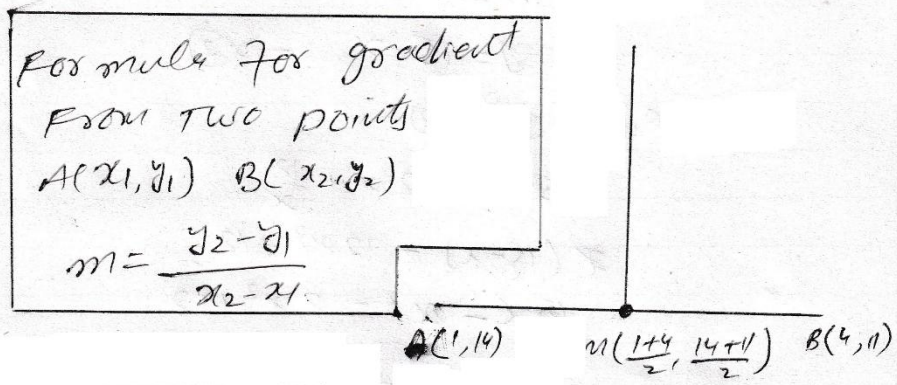
$y = 14$

$A(1, 14)$

(iii)

Formula for gradient from two points $A(x_1, y_1)$ $B(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Gradient of $AB = \frac{11 - 14}{4 - 1}$

$= -\frac{3}{3} = -1$

gradient of the perpendicular bisector of AB = 1

$\therefore m_1 \times m_2 = -1$

$m_2 = -\frac{1}{m_1}$

mid-point of AB $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{1+4}{2}, \frac{4+1}{2}\right)$

$= (5/2, 5/2)$

Formula

Equation of line from one point $P(x_1, y_1)$ and gradient m

$y - y_1 = m(x - x_1)$

Equation of perpendicular bisector of AB

$y - y_1 = m(x_1 - x)$

$y - \frac{5}{2} = 1\left(x - 5/2\right)$

$y - x = \frac{5}{2} - \frac{5}{2}$

$= \frac{0}{2} = 0$

$y - x = 0$