

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (16)

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The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k .

It is now given that $k=2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve.

Solution

Equation of curve
 $y = 2x^2 + kx + k - 1 \rightarrow \textcircled{1}$

(a) Equation of line
 $y = 2x + 3 \rightarrow \textcircled{2}$

Solving $\textcircled{1}$ & $\textcircled{2}$
 $2x^2 + kx + k - 1 = 2x + 3$
 $2x^2 + kx + k - 1 - 2x - 3 = 0$
 $2x^2 + (k-2)x + k-4 = 0$
 As line is tangent to the curve
 $b^2 - 4ac = 0$
 $\Rightarrow (k-2)^2 - 4(2)(k-4) = 0$
 $k^2 - 4k + 4 - 8(k-4) = 0$
 $k^2 - 4k + 4 - 8k + 32 = 0$
 $k^2 - 12k + 36 = 0$
 $(k-6)^2 = 0$
 $\Rightarrow k = 6$

(b) Given $k=2$
 put $k=2$ in $\textcircled{1}$
 $y = 2x^2 + 2x + 2 - 1$

$$y = 2x^2 + 2x + 1$$

$$= 2 \left(x^2 + x + \frac{1}{2} \right)$$

$$= 2 \left[x^2 + 2 \left(\frac{1}{2} \right) x + \frac{1}{2} \right]$$

$$= 2 \left[x^2 + 2 \left(\frac{1}{2} \right) x + \frac{1}{2} \right]$$

$$= 2 \left[x^2 + 2 \left(\frac{1}{2} \right) x + \left(\frac{1}{2} \right)^2 - \left(\frac{1}{2} \right)^2 + \frac{1}{2} \right]$$

$$= 2 \left(x + \frac{1}{2} \right)^2 - 2 \times \left(\frac{1}{2} \right)^2 + 2 \times \frac{1}{2}$$

$$= 2 \left(x + \frac{1}{2} \right)^2 - 2 \times \frac{1}{4} + 1$$

$$= 2 \left(x + \frac{1}{2} \right)^2 - \frac{1}{2} + 1$$

$$= 2 \left(x + \frac{1}{2} \right)^2 + \frac{1}{2}$$

coordinates of the vertex is $\left(-\frac{1}{2}, \frac{1}{2} \right)$.