Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (17)

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The equation of a curve is $y = 2x^2 + m(2x + 1)$,where m is a constant,

And the equation of a line is $y = 6x + 4$. Show that, for all values of m, the line intersects the curve at two distinct points.

Solution

Equation of curve

$$y = 2x^2 + m(2x+1) \rightarrow ①$$

Equation of line

$$y = 6x + 4 \rightarrow ②$$

solving ① & ②

$$6x + 4 = 2x^2 + m(2x+1)$$

$$2x^2 + 2mx + m - 6x - 4 = 0$$

$$2x^2 + (2m-6)x + (m-4) = 0$$

By discriminant

$$a = 2, \quad b = 2m - 6, \quad c = m - 4$$

$$b^2 - 4ac = (2m-6)^2 - 4(2)(m-4)$$

$$= 4m^2 - 24m + 36 - 8(m-4)$$

$$= 4m^2 - 24m + 36 - 8m + 32$$

$$= 4m^2 - 32m + 68$$

$$= 4(m^2 - 8m) + 68$$

$$= 4(m^2 - 2(4)m) + 68$$

$$= 4(m^2 - 2(4)m + (4)^2 - (4)^2) + 68$$

$$= 4(m^2 - 2(4)m + (4)^2) - 4 \times (4)^2 + 68$$

$$= 4(m-4)^2 - 64 + 68$$

$= 4(m-4)^2+4$

As $4>0$ and $(m-4)^2$ always greater than 0 due to square

$\Rightarrow 4(m-4)^2+4>0$ for all m

$\Rightarrow b^2-4ac>0$

So The line intersects The curve at two distinct points