

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

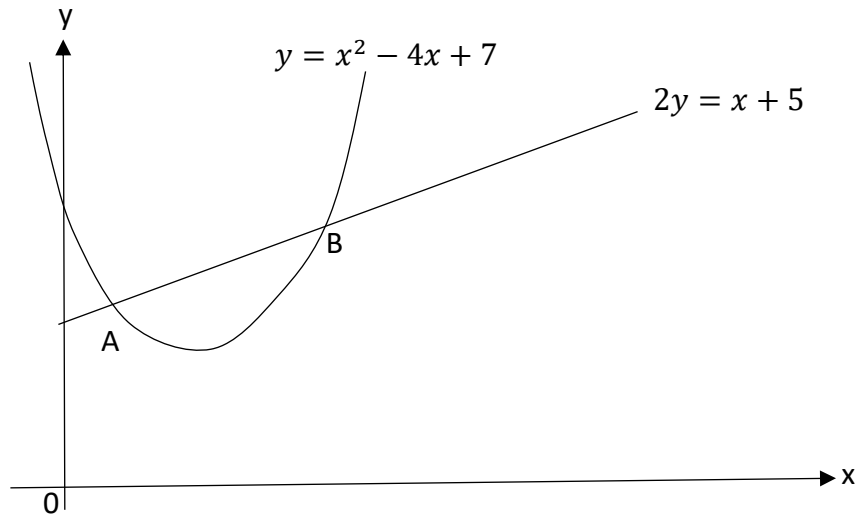
Topic 1-Quadratics

Question No (6)

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Question No (6)

(i) The diagram shows the line $2y = x + 5$ and the curve $y = x^2 - 4x + 7$, which intersect at the points A and B. Find

(a) the x-coordinates of A and B,

(b) the equation of the tangent to the curve at B,

(c) the acute angle, in degrees correct to 1 decimal place, between this tangent and the line $2y = x + 5$.

(ii) Determine the set of values of k for which the line $2y = x + k$ does not intersect the curve

$$y = x^2 - 4x + 7$$

Solution

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①

Equation of line

$$2y = x + 5 \rightarrow \textcircled{1}$$

Equation of curve

$$y = x^2 - 4x + 7 \rightarrow \textcircled{2}$$

From ①

$$y = \frac{x+5}{2} \rightarrow \textcircled{3}$$

solving ② & ③

$$\frac{x+5}{2} = x^2 - 4x + 7$$

$$x+5 = 2(x^2 - 4x + 7)$$

$$x+5 = 2x^2 - 8x + 14$$

$$2x^2 - 8x - x - 5 + 14 = 0$$

$$2x^2 - 9x + 9 = 0$$

Factorize

$$2x^2 - 6x - 3x + 9 = 0$$

$$2x(x-3) - 3(x-3) = 0$$

$$(x-3)(2x-3) = 0$$

$$x-3=0, 2x-3=0$$

$$x=3, x=\frac{3}{2}$$

Hence x -coordinates are

A is $x = 3/2$ and B is $x = 3$

(b)

Equation of line

$$2y = x + 5$$

As $x = 3$ (part (a))

$$2y = 3 + 5$$

$$y = 8/2 = 4$$

B(3, 4)

Equation of curve

$$y = x^2 - 4x + 7$$

differentiate w.r.t x

$$\frac{dy}{dx} = 2x - 4$$

at B(3, 4)

$$\frac{dy}{dx} = 2(3) - 4$$

$$= 6 - 4$$

$$= 2$$

Knowledge showing
Equation of tangent when
gradient and point $P(x_1, y_1)$ is
given

$$y - y_1 = \frac{dy}{dx} (x - x_1)$$

$$\frac{dy}{dx} = 2, \quad B(3, 4)$$

Equation of tangent at $B(3, 4)$

$$y - 4 = 2(x - 3)$$

$$y - 4 = 2x - 6$$

$$y = 2x - 6 + 4$$

$$y = 2x - 2$$

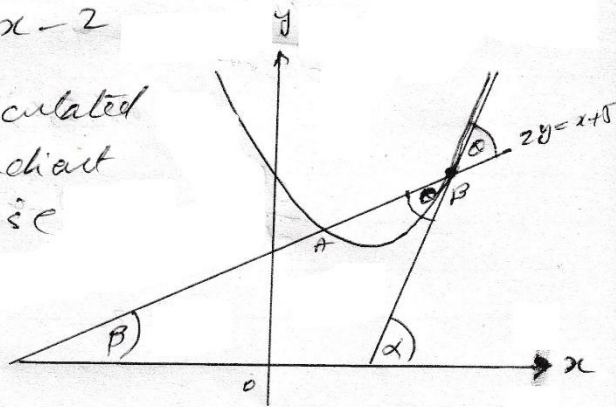
$$y = 2x - 2$$

③ As we have calculated
in part (a) gradient
of tangent is 2 i.e.

$$\frac{dy}{dx} = 2$$

$$\Rightarrow \tan \alpha = 2$$

$$\alpha = \tan^{-1}(2) = 63.5^\circ$$



let α be the angle between the line and tangent line at B.

if β is the angle made by given

line $2y = x + 5$ with x -axis then gradient of line

$$2y = x + 5 \text{ is}$$

$$y = \frac{1}{2}x + \frac{5}{2}$$

compare with

$$y = mx + c$$

$$\Rightarrow m = \frac{1}{2}$$

$$\Rightarrow \tan \beta = \frac{1}{2}$$

$$\beta = 26.5^\circ$$

knowledge sharing

Exterior angle equal to
The sum of two interior
angles

$$\alpha = \theta + \beta$$

From the diagram

$$\alpha = \theta + \beta$$

$$\begin{aligned}\theta &= \alpha - \beta \\ &= 63.5^\circ - 26.5^\circ \\ \theta &= 37^\circ\end{aligned}$$

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Equation of line

$$2y = x + k$$

$$y = \frac{x+k}{2} \rightarrow (1)$$

Equation of curve

$$y = x^2 - 4x + 7 \rightarrow (2)$$

from (1) & (2)

$$\frac{x+k}{2} = x^2 - 4x + 7$$

$$x+k = 2(x^2 - 4x + 7)$$

$$x+k = 2x^2 - 8x + 14$$

$$2x^2 - 8x - x + 14 - k = 0$$

$$2x^2 - 9x + (14 - k) = 0$$

As the line does not intersect curve, so

$$b^2 - 4ac < 0$$

$$(-9)^2 - 4(2)(14 - k) < 0$$

$$81 - 8(14 - k) < 0$$

$$81 - 112 + 8k < 0$$

$$-31 + 8k < 0$$

$$8k < 31$$

$$k < \frac{31}{8}$$