Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (7)

Question No (7)

A curve has equation $y = k\,x^2 + 1$ and a line has equation $y = kx$, where k is a non-zero constant.

(i) Find the set of values of k for which the curve and the line have no common points.

(ii) State the value of k for which the line is a tangent to the curve and, for this case, find the coordinates of the point where the line touches the curve

Solution

(i) Equation of curve

$$y = kx^2 + 1 \longrightarrow (1)$$

Equation of line

$$y = kx \longrightarrow (2)$$

put $y = kx$ in (1)

$$kx = kx^2 + 1$$

$$kx^2 - kx + 1 = 0$$

As line and curve has no common points, so

$$b^2 - 4ac < 0$$

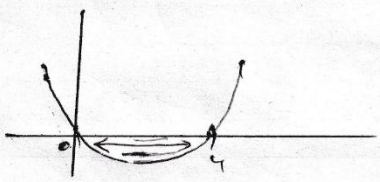
$$\Rightarrow (-k)^2 - 4(k)(1) < 0$$

$$k^2 - 4k < 0$$

$$k(k-4) < 0$$

As in $kx^2 - kx + 1 = 0$

k is +ve, so it will cup face up



So $0 < k < 4$

(ii) From part (i)

$$kx^2 - kx + 1 = 0 \rightarrow (1)$$

As line is tangent to curve

$$b^2 - 4ac = 0$$

$$\Rightarrow (-k)^2 - 4(k)(1) = 0$$

$$k^2 - 4k = 0$$

$$k(k-4) = 0$$

$$k = 0, \quad k - 4 = 0$$

$$k = 0, \quad k = 4$$

As k is non-zero

$$\Rightarrow k = 4$$

put $k = 4$ in (1)

$$4x^2 - 4x + 1 = 0$$

$$\Rightarrow (2x-1)^2 = 0$$

$$2x - 1 = 0$$

$$x = 1/2$$

put $x = \frac{1}{2}$ in Equation of line

$$y = kx$$

$$y = 4x \qquad \because k = 4$$

$$y = 4(1/2) = 2 \qquad \because x = 1/2$$

so point of porter section is

$\left(\frac{1}{2}, 2\right)$