

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (9)

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Question No (9)

The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k=8$, find the coordinates of the points of intersection of the line and the curve.

(ii) Find the value of k for which the line is a tangent to the curve.

Solution

DATE :-

Equation of curve
 $y^2 + 2x = 13 \rightarrow \textcircled{1}$

Equation of line
 $2y + x = k \rightarrow \textcircled{2}$

$\textcircled{i}$ As $k=8$ Equation $\textcircled{2}$ becomes
 $2y + x = 8$
 $\Rightarrow x = 8 - 2y \rightarrow \textcircled{3}$

put value of x in $\textcircled{1}$
 $y^2 + 2(8 - 2y) = 13$
 $y^2 + 16 - 4y = 13$
 $y^2 - 4y + 16 - 13 = 0$
 $y^2 - 4y + 3 = 0$
 factorize
 $y^2 - 3y - y + 3 = 0$
 $y(y - 3) - 1(y - 3) = 0$
 $(y - 3)(y - 1) = 0$
 $y - 3 = 0, y - 1 = 0$

$$y=3,$$

when $y=3$

Equation (3) becomes

$$x = 8 - 2y$$

$$x = 8 - 2(3)$$

$$= 8 - 6$$

$$x = 2$$

$$(2, 3)$$

$$y=1$$

when $y=1$

Equation (3) becomes

$$x = 8 - 2y$$

$$= 8 - 2(1)$$

$$= 8 - 2$$

$$x = 6$$

$$(6, 1)$$

So points of intersection for line and curve are

$$(2, 3) \quad (6, 1)$$

(ii)

Equation of curve

$$y^2 + 2x = 13 \rightarrow (1)$$

Equation of line

$$2y + x = k$$

$$x = k - 2y \rightarrow (2)$$

put $x = k - 2y$ in (1)

$$y^2 + 2(k - 2y) = 13$$

$$y^2 + 2k - 4y = 13$$

$$y^2 - 4y + 2k - 13 = 0$$

As the line is tangent to curve

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4(1)(2k - 13) = 0$$

$$16 - 4(2k - 13) = 0$$

$$16 - 8k + 52 = 0$$

$$-8k = -16 - 52$$

$$-8k = -68$$

$$k = \frac{-68}{-8}$$

$$k = 8.5$$