Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 1-Quadratics

Question No (9)

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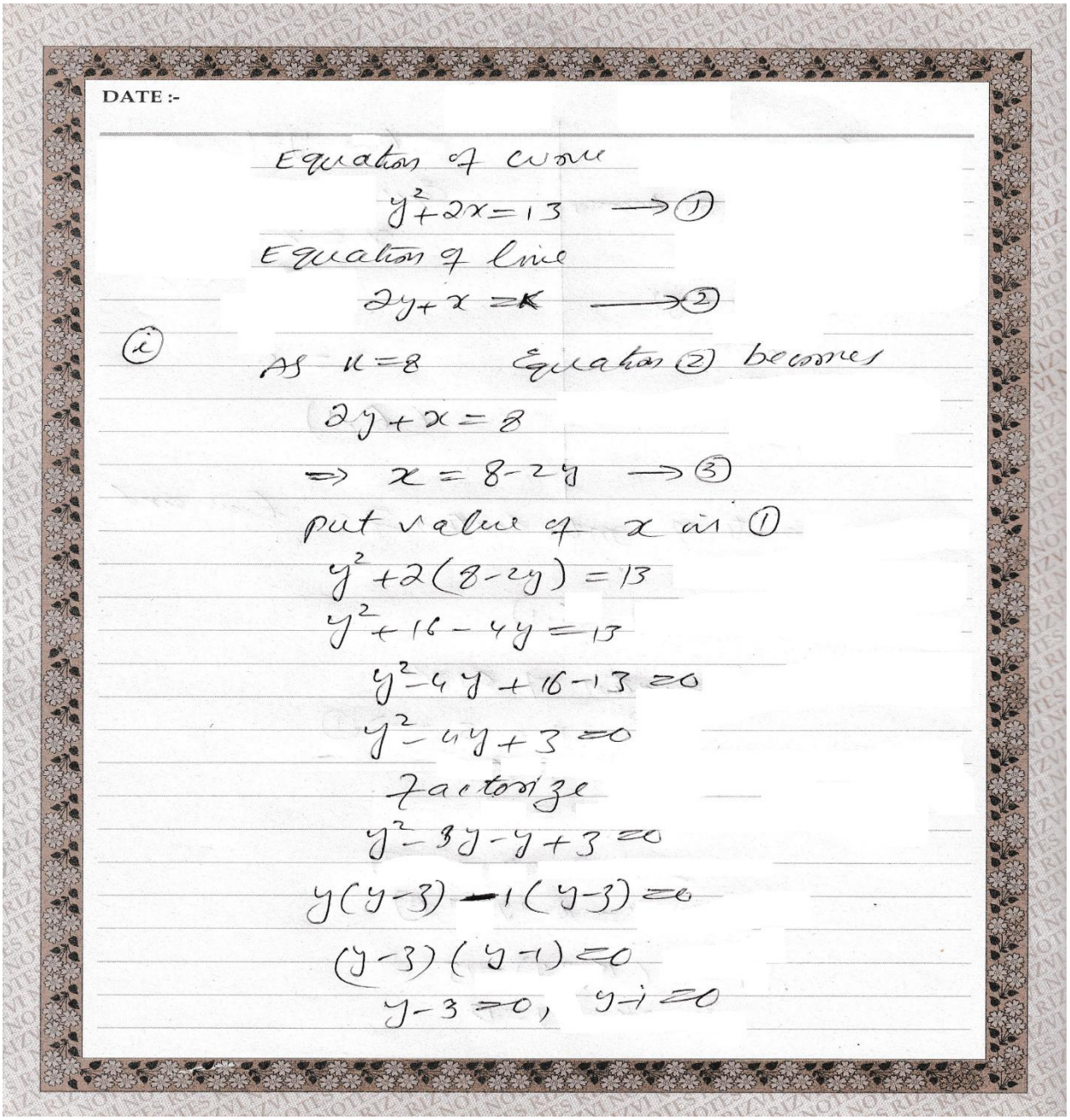
The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where k =8, find the coordinates of the points of intersection of the line and the curve.

(ii) Find the value of k for which the line is a tangent to the curve.

Solution



$y=3$, $y=1$

when $y=3$

Equation ③ becomes

$x = 8-2y$

$x = 8-2(3)$

$= 8-6$

$x = 2$

$(2,3)$

when $y=1$

Equation ③ becomes

$x = 8-2y$

$= 8-2(1)$

$= 8-2$

$x = 6$

$(6,1)$

so points of intersection for line and curve are

$(2,3)$ $(6,1)$

(ii)

Equation of curve

$y^2+2x=13 \longrightarrow ①$

Equation of line

$2y+x=k$

$x = k-2y \longrightarrow ②$

put $x = k-2y$ in ①

$y^2+2(k-2y)=13$

$y^2+2k-4y=13$

$y^2 - 4y + 2k - 13 = 0$

As the line is tangent to curve

$b^2 - 4ac = 0$

$(-4)^2 - 4(1)(2k-13) = 0$

$16 - 4(2k-13) = 0$

$16 - 8k + 52 = 0$

$-8k = -16 - 52$

$-8k = -68$

$k = \frac{-68}{-8}$

$k = 8.5$