

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (1)

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Question No (1)

Functions f and g are defined by

$$f: x \rightarrow k - x \text{ for } x \in \mathbb{R}, \text{ where } k \text{ is constant}$$

$$g: x \rightarrow \frac{9}{x+2} \text{ for } x \in \mathbb{R}, x \neq -2$$

(i) Find the values of k for which the equation $f(x)=g(x)$ has two equal roots and solve the equation $f(x)=g(x)$ in these cases.

(ii) Solve the equation $fg(x)=5$ when $k=6$

(iii) Express $g^{-1}(x)$ in terms of x .

Solution

(i)

$$\begin{aligned}
 f(x) &= k - x \rightarrow (1) \\
 \text{and } g(x) &= \frac{9}{x+2} \rightarrow (2) \\
 \text{By the given condition} \\
 f(x) &= g(x) \\
 k - x &= \frac{9}{x+2} \rightarrow (3) \\
 (x+2)(k-x) &= 9 \\
 x(k-x) + 2(k-x) &= 9 \\
 kx - x^2 + 2k - 2x &= 9 \\
 -x^2 + (k-2)x + 2k-9 &= 0 \\
 \text{As } f(x)=g(x) \text{ has equal roots, so} \\
 b^2 - 4ac &= 0 \\
 (k-2)^2 - 4(-1)(2k-9) &= 0 \\
 k^2 - 4k + 4 + 4(2k-9) &= 0 \\
 k^2 - 4k + 4 + 8k - 36 &= 0 \\
 k^2 + 4k - 32 &= 0 \\
 \text{By factorization} \\
 k^2 + 8k - 4k - 32 &= 0
 \end{aligned}$$

$$k(k+8) - 4(k+8) = 0$$

$$(k+8)(k-4) = 0$$

$$k+8=0, \quad k-4=0$$

$$k=-8, \quad k=4$$

when $k=-8$

Equation ③ becomes

$$-8-x = \frac{9}{x+2}$$

$$(x+2)(-8-x) = 9$$

$$x(-8-x) + 2(-8-x) = 9$$

$$-8x - x^2 - 16 - 2x = 9$$

$$-x^2 - 10x - 16 - 9 = 0$$

$$-x^2 - 10x - 25 = 0$$

$$-1(x^2 + 10x + 25) = 0$$

$$x^2 + 10x + 25 = 0$$

Factorization

$$x^2 + 5x + 5x + 25 = 0$$

$$x(x+5) + 5(x+5) = 0$$

$$(x+5)(x+5) = 0$$

$$(x+5)^2 = 0$$

$$\Rightarrow x+5=0$$

$$x = -5$$

when $k=4$

Equation ③ becomes

$$4-x = \frac{9}{x+2}$$

$$(4-x)(x+2) = 9$$

$$4(x+2) - x(x+2) = 9$$

$$4x + 8 - x^2 - 2x = 9$$

$$-x^2 + 2x + 8 - 9 = 0$$

$$-x^2 + 2x - 1 = 0$$

$$-(x^2 - 2x + 1) = 0$$

$$x^2 - 2x + 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1 = 0$$

$$x = 1$$

(ii)

Given equation

$$f(g(x)) = 5$$

$$f\left(\frac{9}{x+2}\right) = 5$$

$$K - \frac{9}{x+2} = 5 \quad \because f(x) = K - x$$

$$6 - \frac{9}{x+2} = 5 \quad \because K = 6$$

$$6(x+2) - 9 = 5(x+2)$$

$$6x + 12 - 9 = 5x + 10$$

$$6x - 5x = 10 + 9 - 12$$

$$x = 7$$

(iii)

$$g(x) = \frac{9}{x+2}$$

$$y = \frac{9}{x+2} \quad \because g(x) = y$$

$$y(x+2) = 9$$

$$yx + 2y = 9$$

$$yx = 9 - 2y$$

$$x = \frac{9 - 2y}{y}$$

$$g^{-1}(y) = \frac{9 - 2y}{y}$$

$$\because x = g^{-1}(y)$$

$$\Rightarrow \bar{g}'(x) = \frac{9-2x}{x} \quad \text{replacing } y \text{ by } x.$$

