Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (2)

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The function f is defined by

$$f: x \to x^2 - 3x \; for \; x \in \mathbb{R}$$

(i) Find the set of values of x for which $f(x) > 4$.

(ii) Express $f(x)$ in the form $(x-a)^2 - b$, stating the values of a and b.

(iii) Write down the range of f.

(iv) State, with a reason, whether f has an inverse.

The function g is defined by $g: x \to x - 3\sqrt{x} \; for \; x \geq 0$

(v) Solve the equation $g(x)$=10.

Solution

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$f: x \to x^2 - 3x$

$f(x) = x^2 - 3x$

(i)

given condition $f(x) > 4$

$\Rightarrow x^2 - 3x > 4$ $\quad$ $\because f(x) = x^2 - 3x$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$ $\quad$ factorize

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

critical values

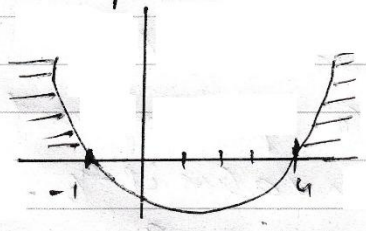
$x - 4 = 0, \; x + 1 = 0$

$x = 4, \; x = -1$

$\because (x^2 - 3x - 4) > 0$

As $a = 1 > 0$

so up face up



shaded area covering $x < -1$ and $x > 4$

so set of values are $x < -1$ and $x > 4$.

(ii)

$$f(x) = x^2 - 3x$$

$$f(x) = x^2 - \frac{2}{2}(3x)$$

$$= x^2 - 2(3/2)x$$

$$= x^2 - 2(3/2)x + (3/2)^2 - (3/2)^2 \quad \text{completing square}$$

$$= (x - 3/2)^2 - \left(\frac{9}{2}\right)^2$$

$$= (x - 3/2)^2 - \frac{9}{4}$$

So $a = 3/2$, $b = -9/4$

(iii)

From part (ii)

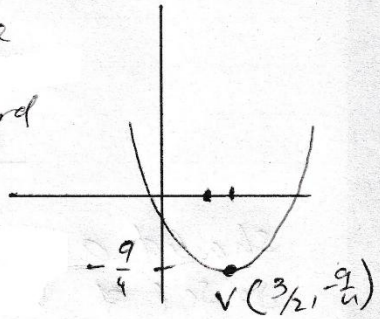
$$f(x) = (x - 3/2)^2 - \frac{9}{4}$$

As $f(x) = x^2 - 3x$

$a = 1 > 0$, so cup face up and it has minimum value

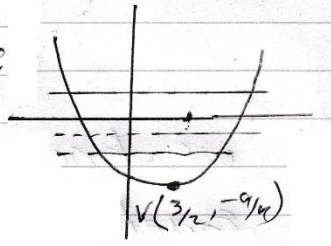
since parabola opens upward and its minimum value is $-9/4$

so range is $f(x) \geq -\frac{9}{4}$



(iv) f has no inverse because horizontal line test fail

Horizontal line cut the graph at two points.



(v) $g: x \to x - 3\sqrt{x}$ for $x \geq 0$

$g(x) = x - 3\sqrt{x}$

$10 = x - 3\sqrt{x}$ i.e. $g(x) = 10$

$10 - x = -3\sqrt{x}$

Squaring both sides

$(10-x)^2 = (3\sqrt{x})^2$

$(10)^2 - 2(10)(x) + (-x)^2 = 9(x)$

$100 - 20x + x^2 = 9x$

$x^2 - 20x - 9x + 100 = 0$

$x^2 - 29x + 100 = 0$

By factorization

$x^2 - 25x - 4x + 100 = 0$

$$x(x-25)-4(x-25)=0$$

$$(x-25)(x-4)=0$$

$$x-25=0, \quad x-4=0$$

$$x=25, \quad x=4$$

we know that in case of radical Equation, we need to verify the solution.

$$g(x)=x-3\sqrt{x} \quad x\geq 0$$

at $x=4$

$$g(4)=4-3(\sqrt{4})$$

$$=4-3(2)=4-6=-2<0$$

As $x\geq 0$, so $x=4$ is not solution.

at $x=25$

$$g(x)=x-3\sqrt{x}$$

$$g(25)=25-3(\sqrt{25})$$

$$=25-3(5)$$

$$=25-15$$

$$=10>0$$

so $x=25$