Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (3)

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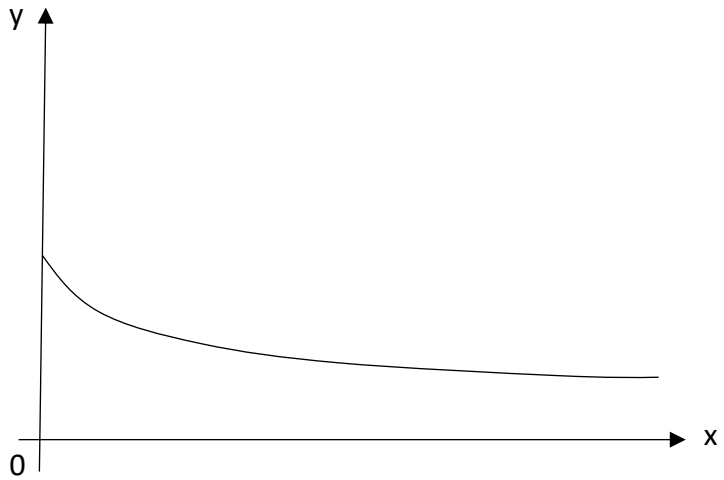

The diagram shows the graph of $y = f(x)$ $where\ f: x \to \frac{6}{2x+3}$ $for\ x \geq 0$.

(i) Find an expression, in terms of x , for $f'(x)$ and explain how your answer shows that f is a decreasing function.

(ii) Find an expression, in terms of x , for $f^{-1}(x)$ and find the domain of f^{-1} .

(iii) Copy the diagram and, on your copy, sketch the graph of $y = f^{-1}(x)$, making clear the relationship between the graphs.

The function g is defined by $g: x \to \frac{1}{2}\ x\ for\ x \geq 0$.

(iv) Solve the equation $fg(x) = \frac{3}{2}$.

Solution

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$$f: x \to \frac{6}{2x+3} \qquad x \geq 0$$

$$f(x) = \frac{6}{2x+3}$$

(i)

$$f(x) = \frac{6}{2x+3}$$

$$f(x) = 6(2x+3)^{-1}$$

differentiate w.r.t x

$$f'(x) = 6(-1(2x+3)^{-1-1} \frac{d}{dx}(2x+3)$$

$$= -6(2x+3)^{-2}(2)$$

$$f'(x) = \frac{-12}{(2x+3)^2}$$

As $f'(x) < 0$ for all values of x, so $f(x)$ is decreasing function.

(ii)

As $f(x) = \frac{6}{2x+3}$

$y = \frac{6}{2x+3}$ $\quad \because f(x) = y$

$y(2x+3) = 6$

$$2xy + 3y = 6$$

$$2xy = 6 - 3y$$

$$x = \frac{6-3y}{2y}$$

$$= \frac{6}{2y} - \frac{3y}{2y}$$

$$x = \frac{3}{y} - \frac{3}{2}$$

$$\bar{f}^{1}(y) = \frac{3}{y} - \frac{3}{2} \qquad x = \bar{f}^{1}(y)$$

$$\bar{f}^{1}(x) = \frac{3}{x} - \frac{3}{2}$$

As $f(x) = \frac{6}{2x+3}$

at $x = 0$

$$f(0) = \frac{6}{2(0)+3}$$

$$= \frac{6}{3} = 2$$

at $x = \infty$

$$f(\infty) = \frac{6}{\infty} = 0$$

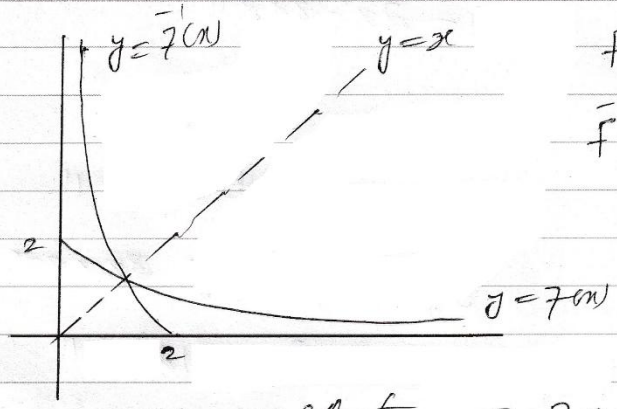
So range of $f(x)$ is $0 < f(x) \leq 2$

$\Rightarrow \bar{f}^{1}(x)$ domain is $0 < x \leq 2$

Knowledge sharing

range of $f(x)$ is equal to the domain of $\bar{f}^{1}(x)$ and vice versa

(iii)



$f(x) = \frac{6}{2x+3}$

$f^{-1}(x) = \frac{3}{x} - \frac{3}{2}$

$f^{-1}(x)$ is the reflection of $f(x)$ in the line $y = x$

(iv)

given equation

$f\,g(x) = 3/2$

$f(\frac{x}{2}) = 3/2$ $\quad \therefore g(x) = x/2$

$\frac{6}{2(x/2)+3} = 3/2$ $\quad = f(x) = \frac{6}{2x+3}$

$\frac{6}{x+3} = 3/2$

$\frac{2 \times 6}{3} = x + 3$

$2 \times 2 = x + 3$ $\quad \Rightarrow x = 4 - 3$

$4 = x + 3$ $\quad x = 1$