Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (6)

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Functions f and g are defined by

$$f : x \to 4x - 2k \quad for\ x \in \mathbb{R}, where\ k\ is\ constant.$$

$$g : x \to \frac{9}{2 - x} \quad for\ x\ \in \mathbb{R}, x \neq 2\ .$$

(i) Find the values of k for which the equation $fg(x) = x$ has two equal roots.

(ii) Determine the roots of the equation $fg(x) = x$ for the values of k found in part (i).

Solution

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$f: x \to 4x - 2k$ for $x \in R$

$f(x) = 4x - 2k$

and $g: x \to \frac{9}{2-x}$ for $x \in R$

$g(x) = \frac{9}{2-x}$

(i)

Given condition

$f(g(x)) = x$ has two equal roots.

$f\left(\frac{9}{2-x}\right) = x$

$4\left(\frac{9}{2-x}\right) - 2k = x$

$4(9) - 2k(2-x) = x(2-x)$

$36 - 4k + 2xk = 2x - x^2$

$36 - 4k + 2xk - 2x + x^2 = 0$

$x^2 + (2k-2)x + (36-4k) = 0 \to (1)$

Has two equal roots

$\Rightarrow b^2 - 4ac = 0$

$(2k-2)^2 - 4(1)(36-4k) = 0$

$(2k)^2 - 2(2k)(2) + (2)^2 - 4(36-4k) = 0$

$4k^2 - 8k + 4 - 144 + 16k = 0$

$4k^2 + 8k - 140 = 0$

$4(K^2+2K-35)=0$

$\Rightarrow K^2+2K-35=0$

factorization

$K^2+7K-5K-35=0$

$K(K+7)-5(K+7)=0$

$(K+7)(K-5)=0$

$K+7=0, \quad K-5=0$

$\therefore \quad K=-7, \quad K=5$

(ii) From equation (i) of part (i)

$x^2+(2K-2)x+(36-4K)=0$

At $K=-7$

$x^2+(2(-7)-2)x+(36-4(-7))=0$

$x^2+(-14-2)x+(36+28)=0$

$x^2-16x+64=0$

factorize

$x^2-8x-8x+64=0$

$x(x-8)-8(x-8)=0$

$(x-8)^2=0 \quad \Rightarrow x-8=0 \Rightarrow x=8$

At $k=5$ equation ① of part(i) becomes

$$x^2+(2(5)-2)x+(36-4(5))=0$$

$$x^2+8x+36-20=0$$

$$x^2+8x+16=0$$

$$(x+4)^2=0$$

$$\Rightarrow x+4=0$$

$$x=-4$$